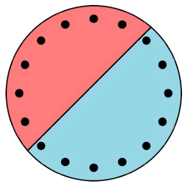


Stresses & Strains

SectionPro Tutorial — Hexagonal, hollow square & U-beam sections under SLS and ULS loads across three normative codes (EC2, NBR-6118, BAEL 91)

BridgeKernel · 2026

Introduction

Given a set of imposed internal forces (N, M_y, M_z), SectionPro computes the strain state $(\varepsilon_0, \kappa_y, \kappa_z)$ that satisfies internal equilibrium, using an iterative solver. From the converged strain state, the software extracts stresses and strains at the extreme fibers of concrete and reinforcement, internal compression and tension forces, and a factor of safety (FS) defined as the ratio of the critical strains to the allowable strain limits.

This article demonstrates the analysis on three geometries and three different standards — a **solid hexagonal section** (Eurocode 2), a **hollow square section** (NBR-6118), and a **custom U-beam with inclined webs** (BAEL 91) — each loaded at both the Serviceability Limit State (SLS, linear material laws) and the Ultimate Limit State (ULS, nonlinear material laws). The load cases are chosen so that some verifications pass (OK) and others fail (KO), showing the behavior of SectionPro when capacity is exceeded across different normative contexts.

Computed results

SectionPro reports three categories of results for each load case:

Stresses & strains

σ_c — Extreme concrete stress
$\sigma_{s,\min}, \sigma_{s,\max}$ — Steel stresses
ε_c — Extreme concrete strain
$\varepsilon_{s,\min}, \varepsilon_{s,\max}$ — Steel strains
FS — Factor of safety
Check — OK / KO

Internal forces

N_c — Compression resultant
N_t — Tension resultant
(x_C, y_C) — Compression centroid
(x_T, y_T) — Tension centroid
z — Internal lever arm

Convergence

N_{iter} — Iterations
Tol — Convergence tolerance
$N_{\text{int}}, M_{z,\text{int}}, M_{y,\text{int}}$ — Internal forces
$\varepsilon_0, \kappa_x, \kappa_y$ — Strain state

Test scenarios

Each section is analyzed at both the Serviceability Limit State (SLS) and the Ultimate Limit State (ULS). In both cases, concrete has zero tensile strength (cracked section). At SLS, the material laws are linear-elastic. At ULS, the laws are nonlinear: concrete follows a parabola-rectangle law and steel follows a bilinear elasto-plastic law. A variety of load cases is introduced: some involve uniaxial combined bending ($N + M_z$), others biaxial bending ($N + M_y + M_z$). Some load cases remain within the section capacity (OK), while others deliberately exceed the allowable limits (KO).

Section	SLS (linear)	ULS (nonlinear)	Biaxial?	Standard
Hexagonal	OK	KO	Yes (ULS)	EC2
Hollow square	OK	OK	Yes	NBR-6118
U-beam	KO	OK	Yes (ULS)	BAEL 91

Solid hexagonal section

Input data

Concrete

- Hexagonal cross section
- Width $B = 2.00$ m
- Minimum thickness $h_1 = 0.60$ m
- Maximum thickness $h_2 = 1.00$ m

Reinforcement

- Uniform spacing 150 mm
- 30 rebars
- Diameter 25 mm
- Cover 50 mm
- 1 layer

Material laws (EC2)

- Concrete C30/37: $f_{ck} = 30$ MPa
- Steel B500B: $f_{yk} = 500$ MPa

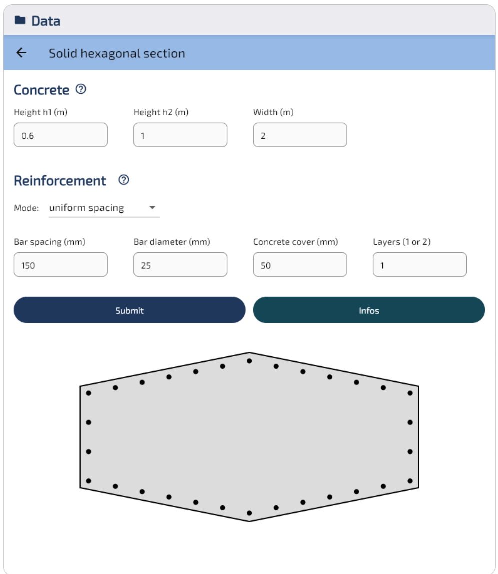


Figure 1: Hexagonal section — geometry and reinforcement.

SLS — Combined bending ($N + M_z$)

Imposed loads: $N = 500$ kN, $M_z = 1000$ kN · m, $M_y = 0$

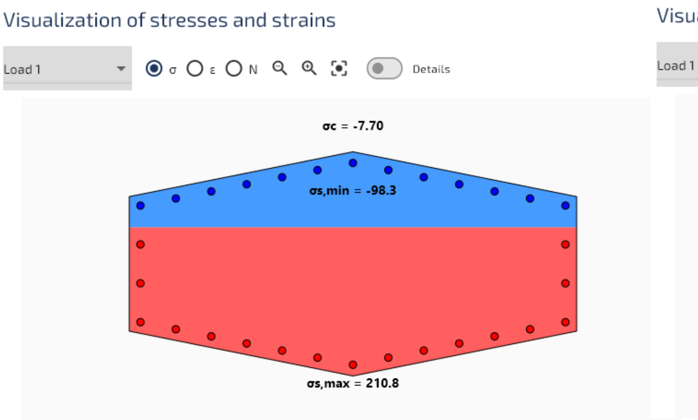


Figure 2: Stress distribution.

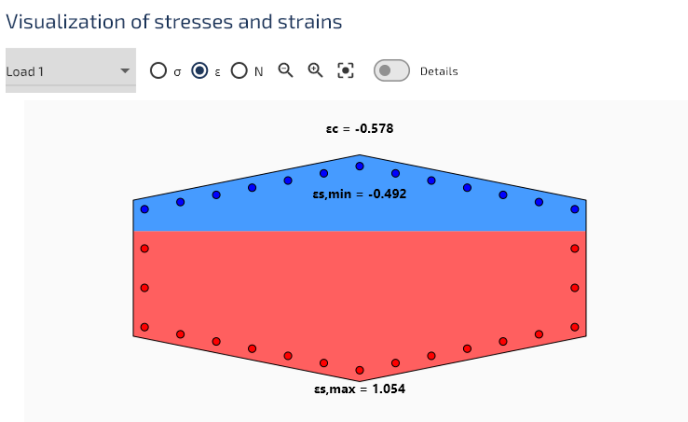


Figure 3: Strain distribution.

Stresses & strains

σ_c	-7.70 MPa
$\sigma_{s,\,\min}$	-98.33 MPa
$\sigma_{s,\,\max}$	210.77 MPa
ε_c	-0.578‰
$\varepsilon_{s,\,\min}$	-0.492‰
$\varepsilon_{s,\,\max}$	1.054‰
FS	0.527
Check	**OK**

Internal forces

N_c	1731.2 kN
N_t	-1231.2 kN
x_C	0.000 m
y_C	0.345 m
x_T	0.000 m
y_T	-0.327 m
z	0.672 m

Convergence

N_{iter}	4
Tol	3.91×10^{-9}
N_{int}	500.0 kN
$M_{z,\text{int}}$	1000.0 kN · m
$M_{y,\text{int}}$	0.0 kN · m
ε_0	0.281×10^{-3}
κ_x	-1.717×10^{-3}
κ_y	0.000×10^{-3}

ULS — Biaxial bending ($N + M_y + M_z$)

Imposed loads: $N = 2000$ kN, $M_z = 3000$ kN · m, $M_y = 1800$ kN · m

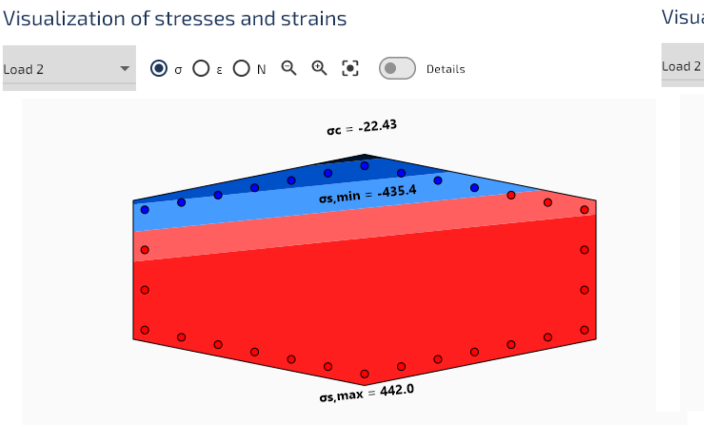


Figure 4: Stress distribution.

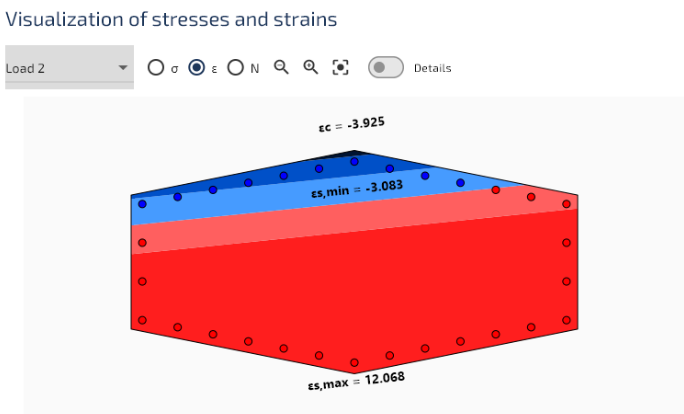


Figure 5: Strain distribution.

Stresses & strains

σ_c	-22.43 MPa
$\sigma_{s,\,\min}$	-435.44 MPa
$\sigma_{s,\,\max}$	441.98 MPa
ε_c	-3.925‰
$\varepsilon_{s,\,\min}$	-3.083‰
$\varepsilon_{s,\,\max}$	12.068‰
FS	1.121
Check	**KO**

Internal forces

N_c	5871.9 kN
N_t	-3871.9 kN
x_C	-0.252 m
y_C	0.356 m
x_T	0.083 m
y_T	-0.235 m
z	0.679 m

Convergence

N_{iter}	8
Tol	6.84×10^{-9}
N_{int}	2000.0 kN
$M_{z,\text{int}}$	3000.0 kN · m
$M_{y,\text{int}}$	1800.0 kN · m
ε_0	4.492×10^{-3}
κ_x	-16.834×10^{-3}
κ_y	-1.721×10^{-3}

When FS > 1, the imposed loads exceed the section capacity. Here, the concrete is crushed ($\varepsilon_c = 3.925$ ‰ $> \varepsilon_{cu} = 3.5$‰). Beyond failure, a secant modulus extends the material law to reach a fictitious post-failure equilibrium, quantifying the exceedance (FS $= 1.121$).

Hollow square section

Input data

Concrete

- Hollow square section
- Outer side $a = 2.0$ m
- Wall thickness $t = 0.30$ m

Reinforcement

- Uniform spacing 150 mm
- 64 rebars
- Diameter 20 mm
- Cover 40 mm
- 1 layer per face (inner + outer)

Material laws (NBR-6118)

- Concrete C30: $f_{ck} = 30$ MPa
- Steel: $f_{yk} = 500$ MPa

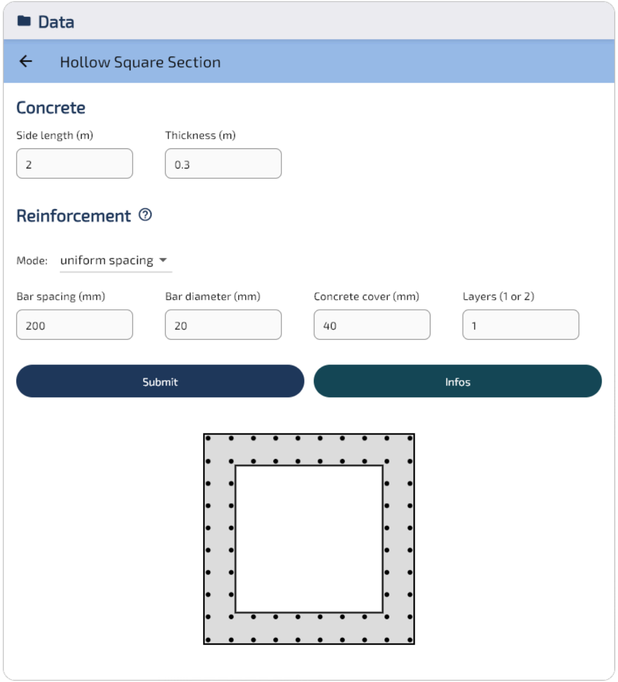


Figure 6: Hollow square section — geometry and reinforcement.

SLS — Biaxial bending $(N + M_y + M_z)$

Imposed loads: $N = -400$ kN, $M_z = 900$ kN · m, $M_y = 400$ kN · m

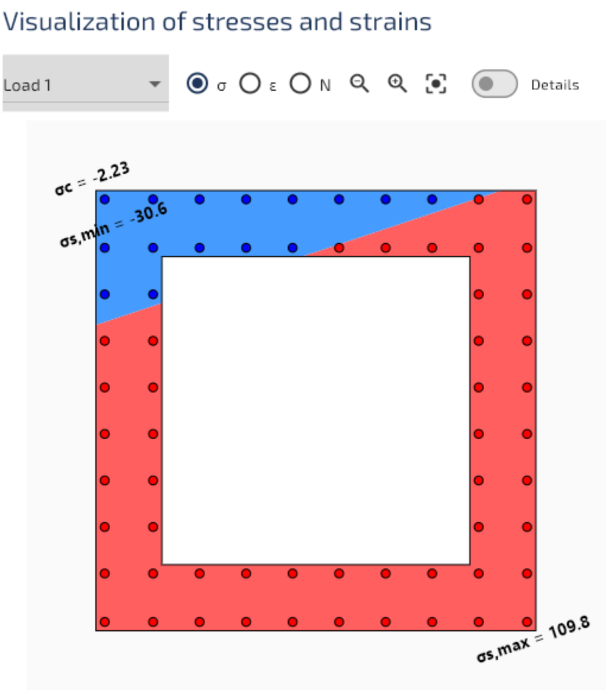


Figure 7: Stress distribution.

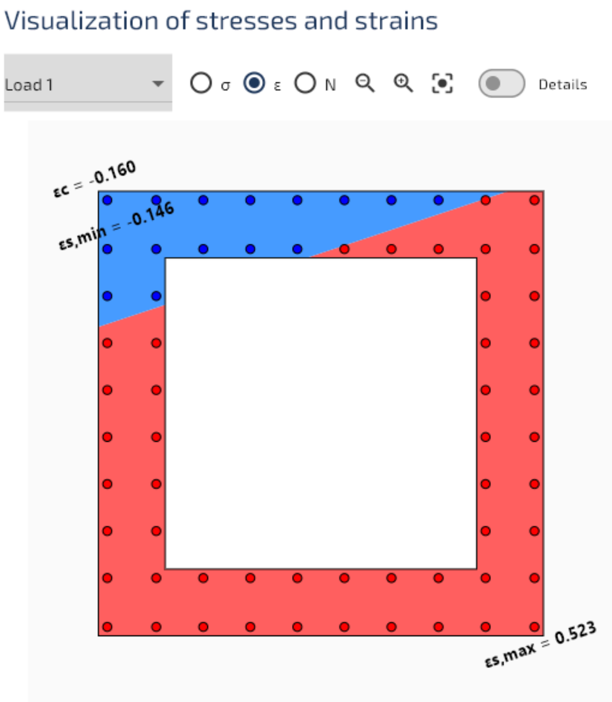


Figure 8: Strain distribution.

Stresses & strains

σ_c	-2.23 MPa
$\sigma_{s,\,\mathrm{min}}$	-30.57 MPa
$\sigma_{s,\,\mathrm{max}}$	109.79 MPa
ε_c	-0.160‰
$\varepsilon_{s,\,\mathrm{min}}$	-0.146‰
$\varepsilon_{s,\,\mathrm{max}}$	0.523‰
FS	0.274
Check	OK

Internal forces

N_c	458.9 kN
N_t	-858.9 kN
x_C	-0.541 m
y_C	0.859 m
x_T	0.177 m
y_T	-0.589 m
z	1.616 m

Convergence

N_{iter}	4
Tol	7.72×10^{-10}
N_{int}	-400.0 kN
$M_{z,\mathrm{int}}$	900.0 kN · m
$M_{y,\mathrm{int}}$	400.0 kN · m
ε_0	0.189×10^{-3}
κ_x	-0.261×10^{-3}
κ_y	-0.087×10^{-3}

ULS — Biaxial bending $(N + M_y + M_z)$

Imposed loads: $N = 0$ kN, $M_z = 6000$ kN · m, $M_y = 6000$ kN · m

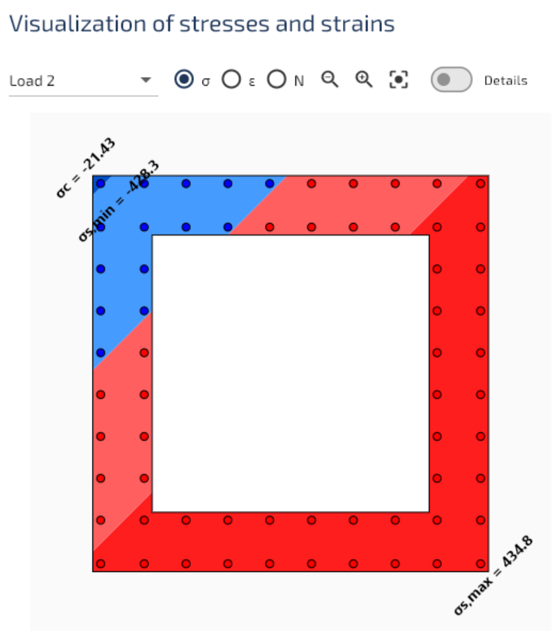


Figure 9: Stress distribution.

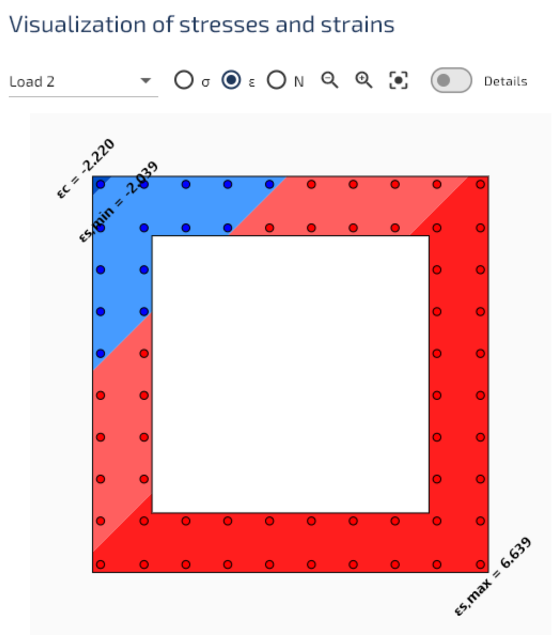


Figure 10: Strain distribution.

Stresses & strains

σ_c	-21.43 MPa
$\sigma_{s,\,\mathrm{min}}$	-428.28 MPa
$\sigma_{s,\,\mathrm{max}}$	434.78 MPa
ε_c	-2.220‰
$\varepsilon_{s,\,\mathrm{min}}$	-2.039‰
$\varepsilon_{s,\,\mathrm{max}}$	6.639‰
FS	0.634
Check	OK

Internal forces

N_c	5800.7 kN
N_t	-1293.2 kN
x_C	-0.751 m
y_C	0.751 m
x_T	-0.085 m
y_T	0.265 m
z	0.824 m

Convergence

N_{iter}	9
Tol	2.35×10^{-7}
N_{int}	0.0 kN
$M_{z,\mathrm{int}}$	6000.0 kN · m
$M_{y,\mathrm{int}}$	6000.0 kN · m
ε_0	2.300×10^{-3}
κ_x	-2.260×10^{-3}
κ_y	-2.260×10^{-3}

Custom section — U-beam

Input data

This section uses the **custom solid geometry** feature. The external contour is defined as a list of XY points, and the reinforcement layout is provided as a table of (x, y, φ) data (position and diameter of each bar). This is the recommended procedure for non-standard geometries that do not fit predefined parametric shapes.

Concrete

- U-beam with inclined webs
- Total height $h = 1.20$ m

Reinforcement

- Uniform spacing 150 mm
- Bottom slab: 11 rebars, diameter 20 mm
- Webs: 49 rebars, diameter 12 mm
- 2 layers per web

Material laws (BAEL 91)

- Concrete: $f_{c28} = 30$ MPa, $\theta = 0.85$
- Steel fe500: $f_e = 500$ MPa

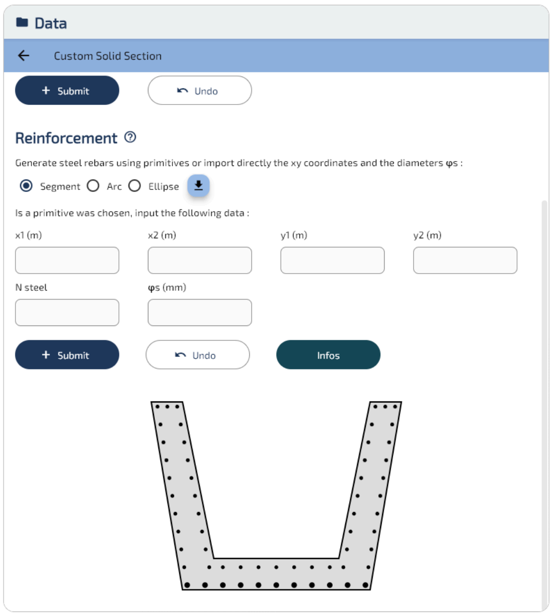


Figure 11: U-beam — geometry and reinforcement.

SLS — Pure bending (M_z)

Imposed loads: $N = 0$ kN, $M_z = 1500$ kN · m, $M_y = 0$

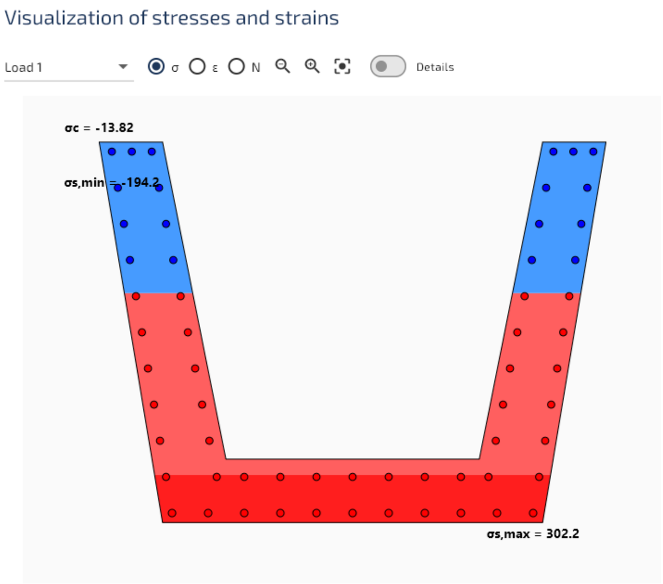


Figure 12: Stress distribution.

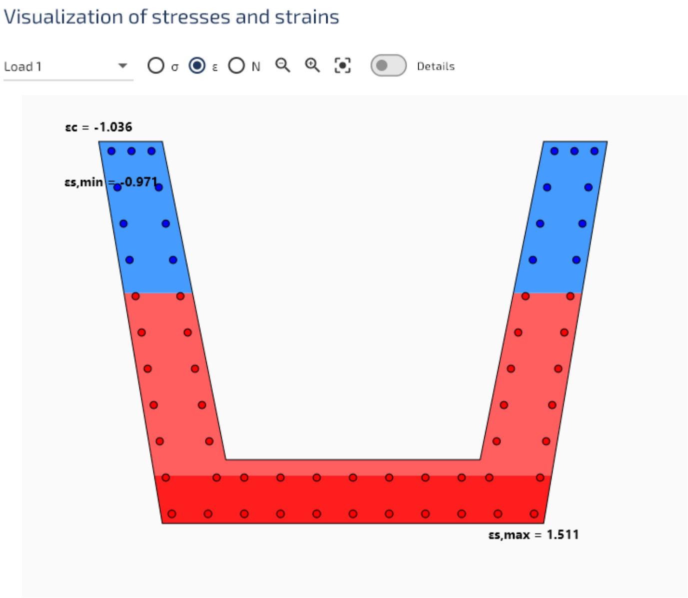


Figure 13: Strain distribution.

Stresses & strains

σ_c	-13.82 MPa
$\sigma_{s,\min}$	-194.19 MPa
$\sigma_{s,\max}$	302.23 MPa
ε_c	-1.036‰
$\varepsilon_{s,\min}$	-0.971‰
$\varepsilon_{s,\max}$	1.511‰
FS	1.209
Check	**KO**

Internal forces

N_c	1593.2 kN
N_t	-1593.2 kN
x_C	0.000 m
y_C	0.571 m
x_T	0.000 m
y_T	-0.371 m
z	0.942 m

Convergence

N_{iter}	4
Tol	2.26×10^{-13}
N_{int}	0.0 kN
$M_{z,\text{int}}$	1500.0 kN · m
$M_{y,\text{int}}$	0.0 kN · m
ε_0	0.543×10^{-3}
κ_x	-2.177×10^{-3}
κ_y	0.000×10^{-3}

At SLS, the check is KO: $\sigma_{s,\max} = 302.2$ MPa exceeds the BAEL allowable stress $\overline{\sigma}_s = 250.0$ MPa (prejudicial cracking, $\eta = 1.60$), giving FS $= 1.209$.

ULS — Biaxial bending ($M_y + M_z$)

Imposed loads: $N = 0$ kN, $M_z = 2000$ kN · m, $M_y = 500$ kN · m

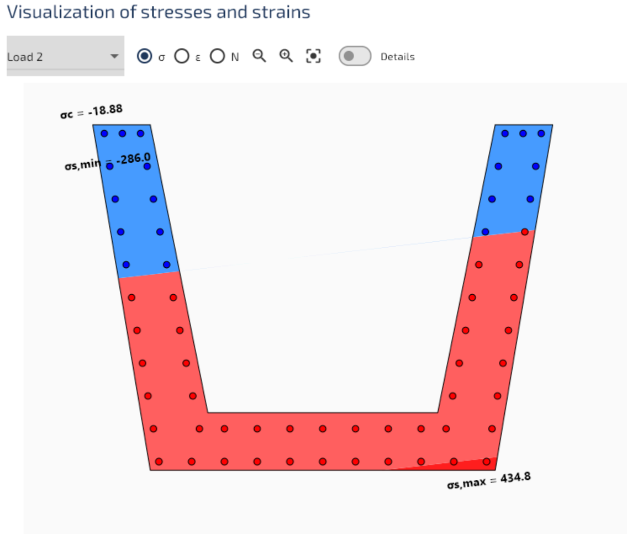


Figure 14: Stress distribution.

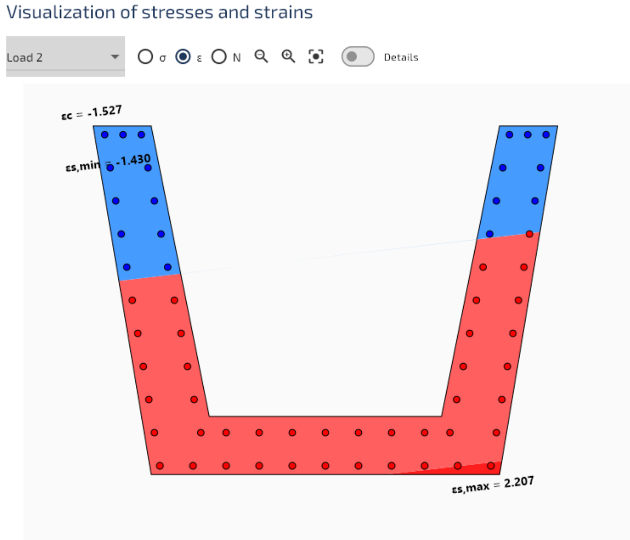


Figure 15: Strain distribution.

Stresses & strains

σ_c	-18.88 MPa
$\sigma_{s,\min}$	-286.01 MPa
$\sigma_{s,\max}$	434.78 MPa
ε_c	-1.527‰
$\varepsilon_{s,\min}$	-1.430‰
$\varepsilon_{s,\max}$	2.207‰
FS	0.436
Check	**OK**

Internal forces

N_c	2158.2 kN
N_t	-2158.2 kN
x_C	-0.192 m
y_C	0.562 m
x_T	0.039 m
y_T	-0.365 m
z	0.955 m

Convergence

N_{iter}	4
Tol	5.50×10^{-9}
N_{int}	0.0 kN
$M_{z,\text{int}}$	2000.0 kN · m
$M_{y,\text{int}}$	500.0 kN · m
ε_0	0.772×10^{-3}
κ_x	-2.811×10^{-3}
κ_y	-0.325×10^{-3}

Results validation

Internal equilibrium check

The imposed loads (N, M_y, M_z) are the **input**. SectionPro finds the strain state $(\varepsilon_0, \kappa_y, \kappa_z)$ by iterative solving, then integrates stresses over the section to obtain the **internal** forces $(N_{\text{int}}, M_{y,\text{int}}, M_{z,\text{int}})$. At convergence, these must match the imposed loads:

$$N_{\text{int}} \approx N \qquad M_{y,\text{int}} \approx M_y \qquad M_{z,\text{int}} \approx M_z$$

Section	Load	N (kN)	N_{int} (kN)	M_z (kN·m)	$M_{z,\text{int}}$ (kN·m)	Δ
Hexagonal	SLS	500.0	500.0	1000.0	1000.0	0.00 %
	ULS	2000.0	2000.0	3000.0	3000.0	0.00 %
Hollow sq.	SLS	−400.0	−400.0	900.0	900.0	0.00 %
	ULS	0.0	0.0	6000.0	6000.0	0.00 %
U-beam	SLS	0.0	0.0	1500.0	1500.0	0.00 %
	ULS	0.0	0.0	2000.0	2000.0	0.00 %

Internal equilibrium is satisfied to machine precision for all six load cases — across three different geometries, three normative codes, and both linear (SLS) and nonlinear (ULS) material laws.

Performance benchmark — 100,000 load cases

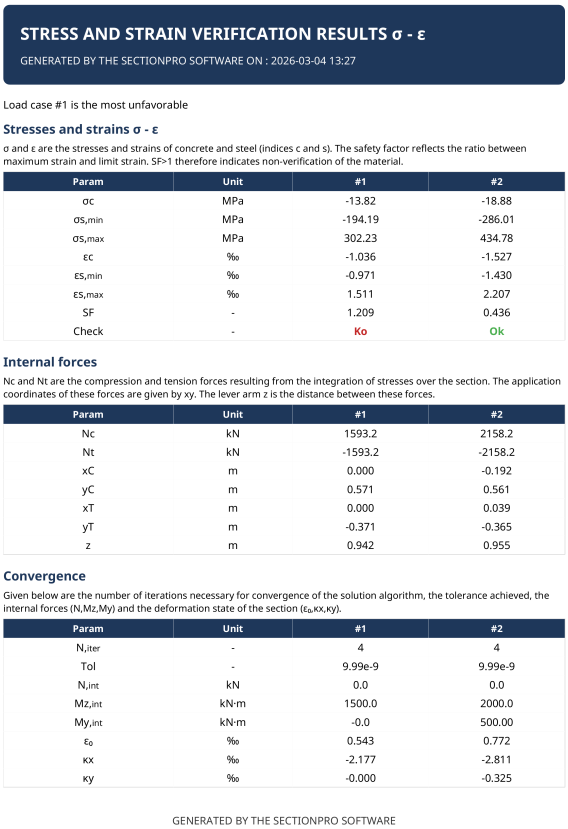
To demonstrate SectionPro's suitability for envelope-based verifications, we run 100,000 load cases on **each of the three sections** defined above. The load cases combine SLS and ULS, linear and nonlinear material laws, uniaxial and biaxial bending, with a mix of OK and KO outcomes. The benchmark measures the pure computation time, excluding UI overhead. Convergence was obtained for all 300,000 load cases.

Metric	Hexagonal	Hollow square	U-beam
Load cases	100,000	100,000	100,000
Computation time	0.173 s	0.304 s	0.260 s
Rate	578,000 loads/s	329,000 loads/s	385,000 loads/s

All three sections complete in under 0.3 seconds — rates of 329,000 to 578,000 load cases per second. This makes SectionPro practical for systematic verifications of large load envelopes generated by finite element software, where thousands of load combinations must be checked in one go.

Export

SectionPro exports results in three formats: **PDF**, **text** (fixed-width columns), and **Excel** (.xlsx). The exported data includes, per load case: stresses and strains, internal forces (with centroids and lever arm), full convergence information, and stress/strain distribution figures.



STRESS AND STRAIN VERIFICATION RESULTS σ - ε

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Load case #1 is the most unfavorable

Stresses and strains σ - ε

σ and ε are the stresses and strains of concrete and steel (indices c and s). The safety factor reflects the ratio between maximum strain and limit strain. SF>1 therefore indicates non-verification of the material.

Param	Unit	#1	#2
σc	MPa	-13.82	-18.88
$\sigma s_{,min}$	MPa	-194.19	-286.01
$\sigma s_{,max}$	MPa	302.23	434.78
εc	‰	-1.036	-1.527
$\varepsilon s_{,min}$	‰	-0.971	-1.430
$\varepsilon s_{,max}$	‰	1.511	2.207
SF	-	1.209	0.436
Check	-	Ko	Ok

Internal forces

Nc and Nt are the compression and tension forces resulting from the integration of stresses over the section. The application coordinates of these forces are given by xy. The lever arm z is the distance between these forces.

Param	Unit	#1	#2
Nc	kN	1593.2	2158.2
Nt	kN	-1593.2	-2158.2
xC	m	0.000	-0.192
yC	m	0.571	0.561
xT	m	0.000	0.039
yT	m	-0.371	-0.365
z	m	0.942	0.955

Convergence

Given below are the number of iterations necessary for convergence of the solution algorithm, the tolerance achieved, the internal forces (N,Mz,My) and the deformation state of the section (ε_0,κx,κy).

Param	Unit	#1	#2
$N_{,iter}$	-	4	4
Tol	-	9.99e-9	9.99e-9
$N_{,int}$	kN	0.0	0.0
$Mz_{,int}$	kN·m	1500.0	2000.0
$My_{,int}$	kN·m	-0.0	500.00
ε_0	‰	0.543	0.772
κx	‰	-2.177	-2.811
κy	‰	-0.000	-0.325

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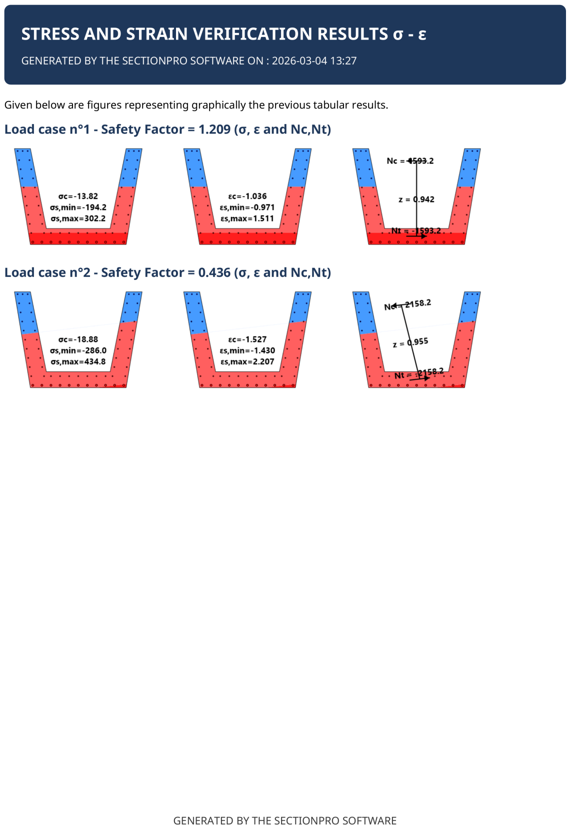
Figure 16: PDF export — page 1: results tables.



STRESS AND STRAIN VERIFICATION RESULTS σ - ε

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Given below are figures representing graphically the previous tabular results.

Load case n°1 - Safety Factor = 1.209 (σ, ε and Nc,Nt)

Load case n°2 - Safety Factor = 0.436 (σ, ε and Nc,Nt)

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Figure 17: PDF export — page 2: figures.

Conclusion

The analysis correctly identifies the load cases that exceed the section capacity, with internal equilibrium satisfied to machine precision in all cases. The three sections span three different normative codes (EC2, NBR-6118, BAEL 91), different geometries (solid, hollow, custom), material laws (linear and nonlinear), and bending states (uniaxial and biaxial).

Section	Load case	Standard	Check	Equilibrium Δ
Hexagonal	SLS (linear)	EC2	OK	0.00 %
	ULS (nonlinear)	EC2	KO	0.00 %
Hollow sq.	SLS (linear)	NBR-6118	OK	0.00 %
	ULS (nonlinear)	NBR-6118	OK	0.00 %
U-beam	SLS (linear)	BAEL 91	KO	0.00 %
	ULS (nonlinear)	BAEL 91	OK	0.00 %

The 100,000-load benchmark shows computation times between 0.17 s and 0.30 s per section, corresponding to rates of 329,000 to 578,000 load cases per second, with convergence obtained for all 300,000 load cases. This makes SectionPro suitable for systematic verifications of load envelopes generated by finite element software.