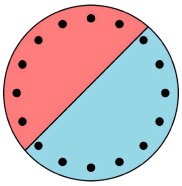


Stiffness Curves

SectionPro Tutorial: moment-curvature diagrams
and flexural stiffness degradation under increasing load

BridgeKernel · 2026

Introduction

Reinforced concrete is not a linear material: its flexural stiffness EI depends on the load level. At low loads, all materials remain in the elastic (steel) or initial tangent (concrete) range of their constitutive laws, so EI is high. As loading increases, the concrete enters the descending branch of its parabola-rectangle law and the steel reaches its yield plateau, causing EI to drop. This degradation matters for estimating realistic displacements, but also more broadly in structural analysis whenever internal forces depend on the distribution of stiffnesses within the model, as in statically indeterminate structures, second-order analysis, and redistribution problems.

SectionPro traces the full section response by fixing two force components and increasing the third (N, M_z, or M_y) from zero to failure. At each step, an iterative equilibrium is solved to find the strain state. This approach also handles biaxial bending: by varying the fixed normal force N, one can observe how the M–χ curve shifts, with compression stiffening the response and tension softening it. Three curves are produced: force–deformation (M–χ), secant stiffness EI_{sec}, and tangent stiffness EI_{tan}. The secant stiffness (slope from the origin to the current point) represents the average stiffness along the loading path, commonly used in iterative FEM analysis. The tangent stiffness (instantaneous slope) gives the exact stiffness for a given load state, used in nonlinear analysis where the stiffness matrix is updated at each step.

The solver also detects *stiffness events*, i.e. key transitions on the constitutive laws: elasticity to plasticity, and rupture. For steel, events can occur in both tension and compression; for concrete, in compression (plastic plateau at ε_{c2} and crushing at ε_{cu2}). Each event is reported with the participant, strain threshold, force level, and corresponding EI_{sec} and EI_{tan}.

Computed results

Curves

M–χ, N–ε_0
EI_{sec}, EI_{tan} vs. load
EA_{sec}, EA_{tan} vs. load

Events table

Participant: concrete / steel
Critical strain threshold
Force, EI or EA at event
Yielding, plastic plateau, crushing

Exports

PDF: stiffness curves + event table
XLS / TXT: full load path + events

Rectangular section (Eurocode 2)

Input data

Concrete

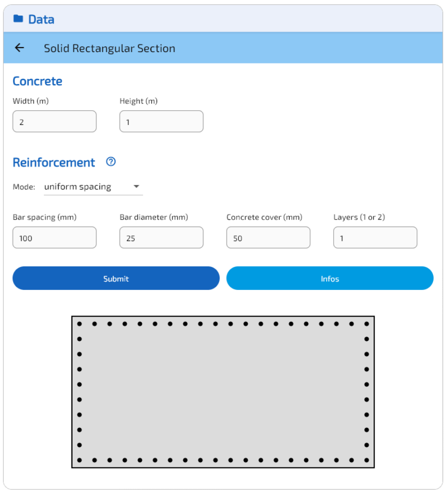
- Solid rectangular cross section
- Width = 2.00 m, Height = 1.00 m

Reinforcement

- 56 bars, uniform spacing 100 mm
- Diameter $\varphi = 25$ mm, cover 50 mm
- Steel ratio $\rho = 1.37\%$

Material laws (EC2)

- Concrete C40/50: $f_{ck} = 40$ MPa
- Steel B500B: $f_{yk} = 500$ MPa



Width (m): 2
Height (m): 1
Mode: uniform spacing
Bar spacing (mm): 100
Bar diameter (mm): 25
Concrete cover (mm): 50
Layers (1 or 2): 1

Figure 1: Rectangular cross section.

The stiffness curve is computed under pure bending: the free component is M_z (curvature χ_z) while $N = 0$ and $M_y = 0$ are held fixed. The limit state is ULS Fundamental ($\gamma_c = 1.50$, $\gamma_s = 1.15$). The curvature is swept from zero to failure, and at each step the corresponding moment and stiffness are computed.

M_z–χ curve and tangent stiffness

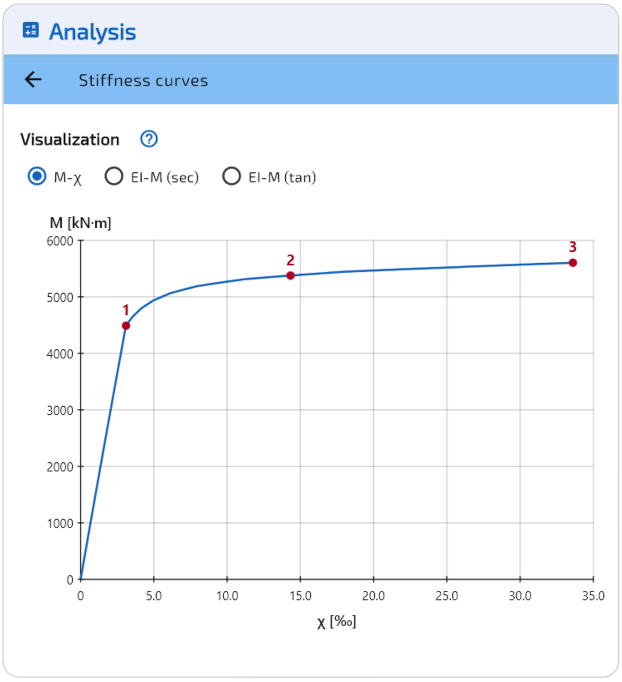


Figure 2: Moment-curvature diagram.

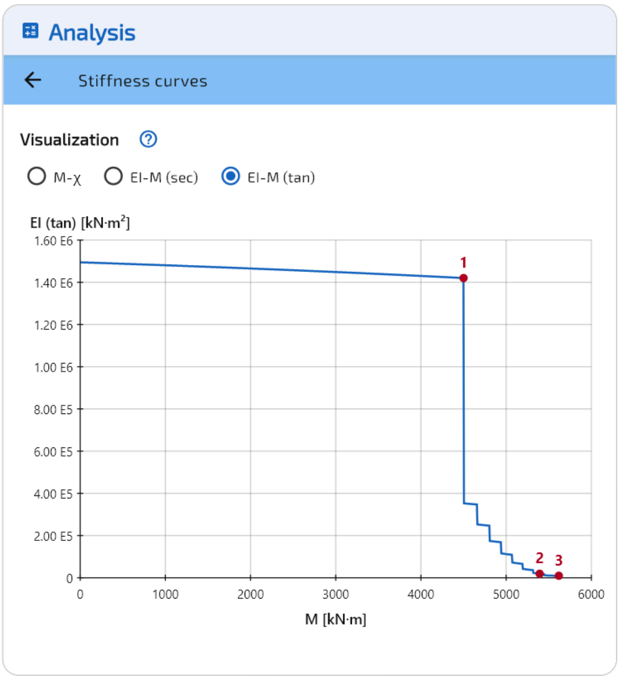
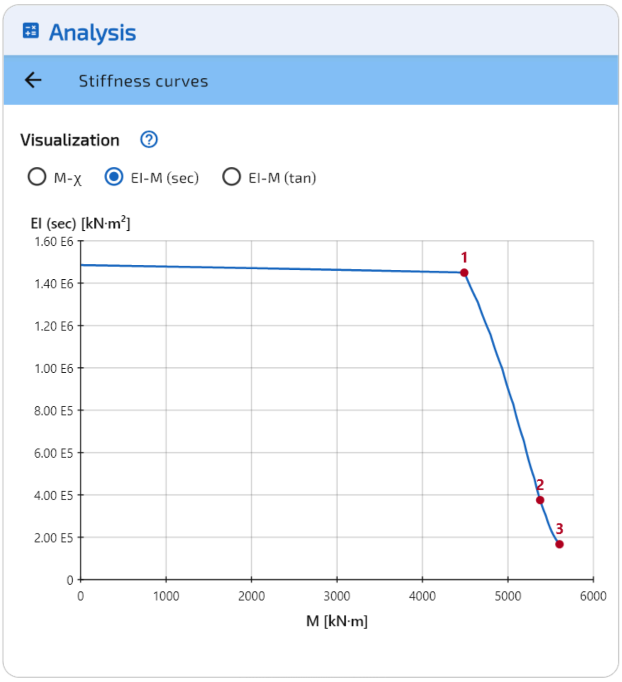


Figure 3: Tangent stiffness EI_{tan}.

The M–χ curve exhibits the classical shape: a steep initial branch where tangent moduli are high, a transition knee at event #1 (steel yielding), and a long plastic plateau where additional curvature produces little extra moment. The ultimate moment is only 25% above the yield moment, but the curvature has increased tenfold.

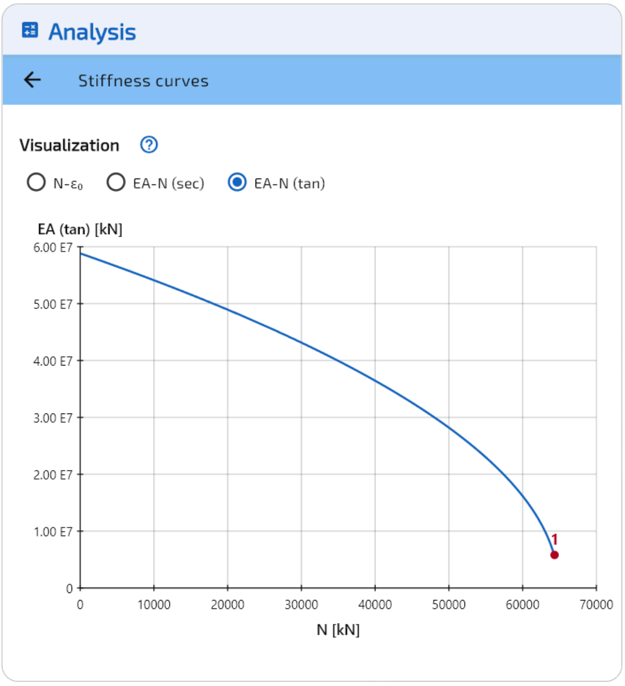
The tangent stiffness remains quasi-constant through the elastic range, then drops sharply at event #1. The drop is abrupt because all bars in the bottom layer share the same y-coordinate and therefore yield simultaneously; this is the main flexural reinforcement, so its loss of stiffness has an immediate effect (EI_{tan} divided by 4 across this single event). When bars are distributed at different depths, the yielding is progressive and the tangent curve shows a staircase pattern instead. Beyond event #2, EI_{tan} falls to near zero, reflecting the almost flat plastic plateau on the M–χ curve.

Secant stiffness EI_{sec} and axial stiffness EA



Visualization ○ M-χ ● EI-M (sec) ○ EI-M (tan)

Figure 4: Secant stiffness EI_{sec}.



Visualization ○ N-ε₀ ○ EA-N (sec) ● EA-N (tan)

Figure 5: Axial stiffness EA.

The secant stiffness remains nearly constant through the elastic range. The drop begins at event #1 (steel yielding), with only a 2% reduction at that point. The steep drop occurs between events #1 and #2, as the steel yields and the concrete enters its plastic plateau. At failure, only about 11% of the initial stiffness remains.

The axial stiffness EA follows a simpler pattern: it decreases as the tangent modulus of the concrete parabola-rectangle law decreases under increasing compressive strain. The curve terminates when the section reaches the ultimate compressive strain of the concrete.

Stiffness events (M–χ)

The solver detects three events along the M–χ curve for this section and limit state:

#	Material	ε_c / ε_s (‰)	χ_z (‰)	M_z (kN·m)	EI_{sec} (kN·m²)	EI_{tan} (kN·m²)
1	Steel	2.174	3.084	4 500	1.459E6	1.420E6
2	Concrete	–2.000	14.764	5 393	3.653E5	1.897E4
3	Concrete	–3.500	34.576	5 618	1.625E5	9.451E3

Event #1 is the onset of steel yielding ($\varepsilon_s = f_{yd}/E_s = 434.8/200\,000 = 2.174\,‰$). Event #2 marks the concrete reaching its plastic plateau strain $\varepsilon_{c2} = 2.000\,‰$. Event #3 is concrete crushing at $\varepsilon_{cu2} = 3.500\,‰$, which terminates the curve.

Hollow oblong section (BAEL 91)

Input data

Concrete

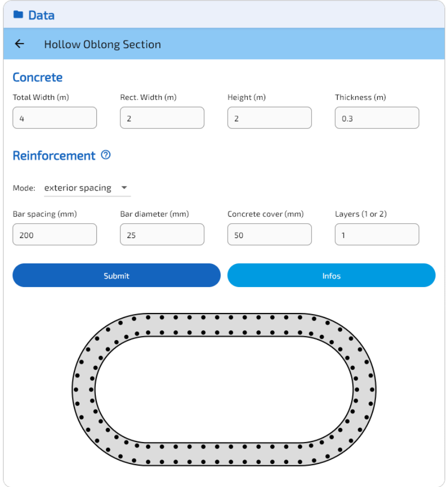
- Hollow oblong cross section
- Total width $= 4.00$ m, Height $= 2.00$ m
- Rectangular width $= 2.00$ m, Thickness $= 0.30$ m

Reinforcement

- 108 bars, exterior spacing 200 mm
- Diameter $\varphi = 25$ mm, cover 50 mm
- Steel ratio $\rho = 1.89\%$

Material laws (BAEL 91)

- Concrete: $f_{c28} = 40$ MPa, $\theta = 0.85$
- Steel: $f_e = 500$ MPa, cracking P



Data

Hollow Oblong Section

Concrete

Total Width (m): 4
Rect. Width (m): 2
Height (m): 2
Thickness (m): 0.3

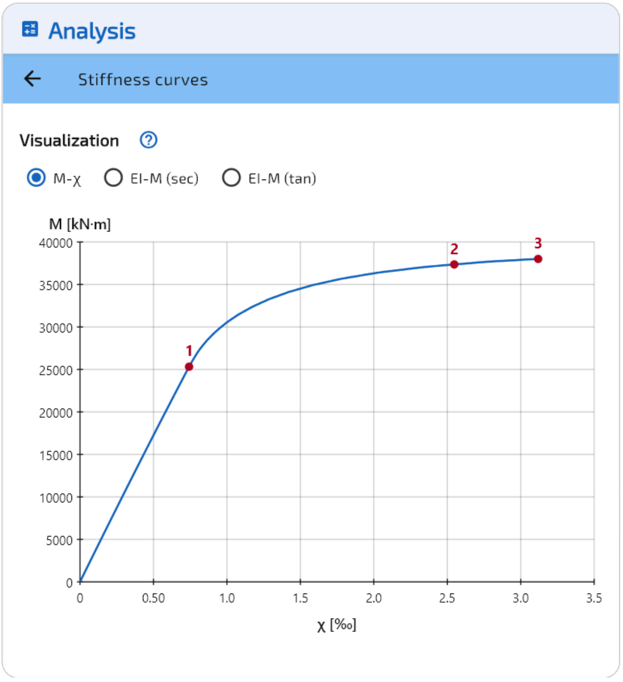
Reinforcement

Mode: exterior spacing
Bar spacing (mm): 200
Bar diameter (mm): 25
Concrete cover (mm): 50
Layers (1 or 2): 1

Figure 6: Hollow oblong section.

The stiffness curve is computed under pure bending about the strong axis: the free component is M_y (curvature χ_y) while $N = 0$ and $M_z = 0$ are held fixed. The limit state is ULS Persistent & Transient. This section is typical of bridge deck cross sections; the large inertia produces a high initial EI and the hollow core amplifies the stiffness drop after cracking.

M_y–χ curve and tangent stiffness



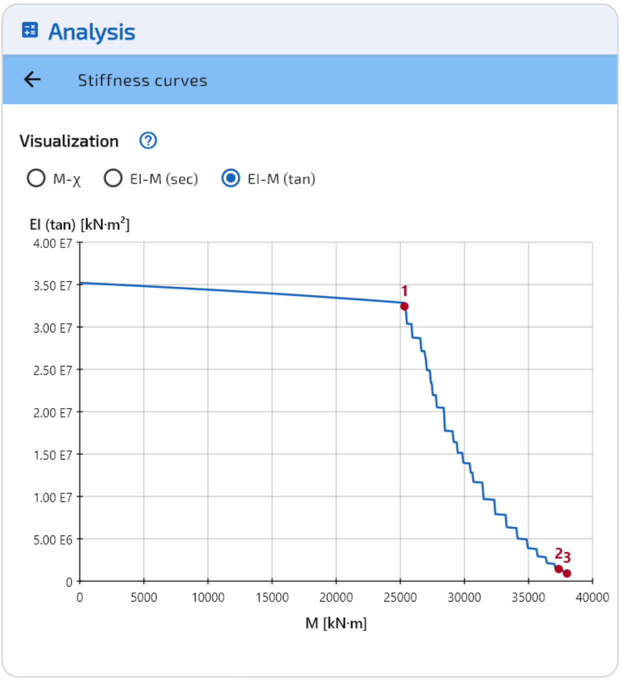
Analysis

Stiffness curves

Visualization

● M-χ ○ EI-M (sec) ○ EI-M (tan)

Figure 7: Moment-curvature diagram.



Analysis

Stiffness curves

Visualization

○ M-χ ○ EI-M (sec) ● EI-M (tan)

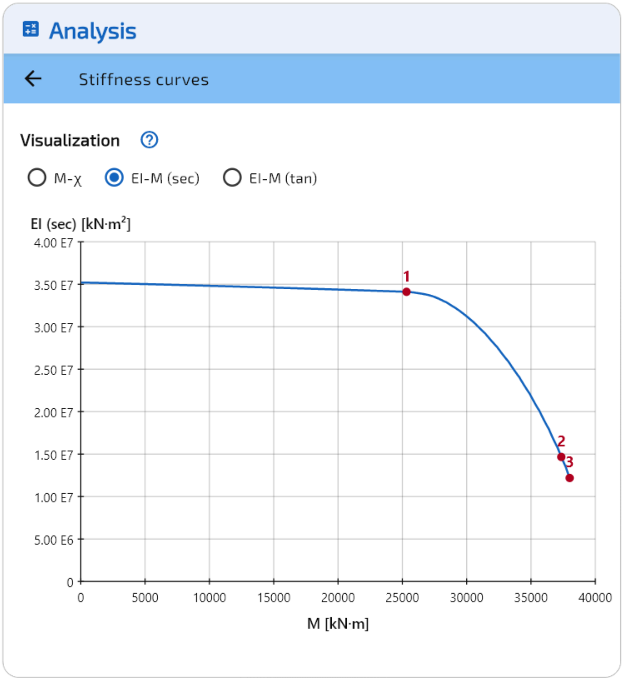
Figure 8: Tangent stiffness $EI_{\tan}$.

The M–χ curve shows that the stiffness degradation begins at event #1 (steel yielding). The ultimate moment is 50% above the yield moment. Here, the curve terminates by steel rupture (event #3) rather than concrete crushing. This is a different failure mode from the rectangular section, where

ε_{cu2} was reached first. Not all events occur for all sections: the failure mode depends on the geometry, reinforcement layout, and material laws.

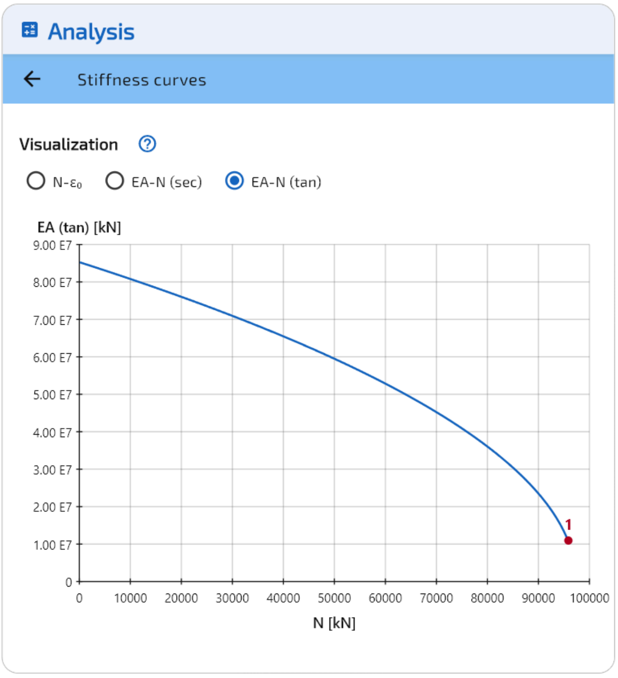
The tangent stiffness remains quasi-constant through the elastic range. The drop begins at event #1, and the staircase pattern (more pronounced here than in the rectangular section) reflects the progressive yielding of individual reinforcement bars around the perimeter. After event #2, EI_{tan} continues to drop, reaching two orders of magnitude below the initial value at failure.

Secant stiffness EI_{sec} and axial stiffness EA



○ M-χ ● EI-M (sec) ○ EI-M (tan)

Figure 9: Secant stiffness EI_{sec}.



○ N-ε₀ ○ EA-N (sec) ● EA-N (tan)

Figure 10: Axial stiffness EA.

The secant stiffness degrades gradually: only a 3% drop at event #1 (steel yielding). The curve steepens after event #2 (concrete plastic plateau), and at failure about 35% of the initial stiffness remains. The smaller relative drop compared to the rectangular section (65% vs. 89%) is typical of hollow sections with high steel ratios.

Stiffness events (M–χ)

#	Material	ε_c / ε_s (‰)	χ_y (‰)	M_y (kN·m)	EI_{sec} (kN·m²)	EI_{tan} (kN·m²)
1	Steel	2.174	0.742	25 324	3.411E7	3.244E7
2	Concrete	–2.000	2.547	37 356	1.466E7	1.439E6
3	Steel	10.000	3.119	38 006	1.219E7	9.154E5

Event #1 is the onset of steel yielding ($\varepsilon_s = 2.174$ ‰). Event #2 marks the concrete reaching its plastic plateau ($\varepsilon_{c2} = 2.000$ ‰). Event #3 is steel rupture at $\varepsilon_{ud} = 10.0$ ‰ (the design ultimate elongation under BAEL), which terminates the curve. Unlike the rectangular section where failure was governed by concrete crushing (ε_{cu2}), this section fails by steel rupture, illustrating that different events are triggered depending on the section geometry and reinforcement layout.

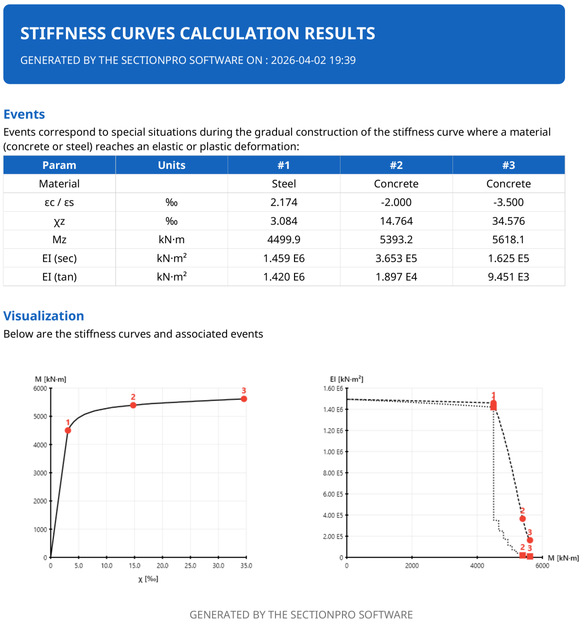
Performance benchmark

Discretisation points	Rectangular EC2 (ms)	Oblong BAEL (ms)
100	5.2	6.5
500	15.5	11.3
1 000 (default)	17.3	19.5
5 000	61.0	60.9

The computation is essentially instantaneous regardless of the number of discretisation points requested: even at 5 000 points, both sections complete in under 61 ms.

Export

SectionPro exports the curve values in PDF, TXT and XLS formats for reuse in external tools. The PDF export also includes visualizations of the curves.



STIFFNESS CURVES CALCULATION RESULTS

GENERATED BY THE SECTIONPRO SOFTWARE ON : 2026-04-02 19:39

Events

Events correspond to special situations during the gradual construction of the stiffness curve where a material (concrete or steel) reaches an elastic or plastic deformation:

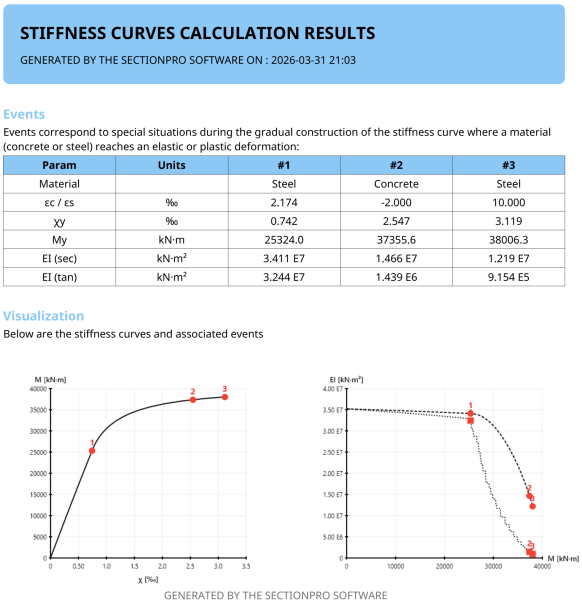
Param	Units	#1	#2	#3
Material		Steel	Concrete	Concrete
εc / εs	‰	2.174	-2.000	-3.500
χz	‰	3.084	14.764	34.576
Mz	kN·m	4499.9	5393.2	5618.1
EI (sec)	kN·m²	1.459 E6	3.653 E5	1.625 E5
EI (tan)	kN·m²	1.420 E6	1.897 E4	9.451 E3

Visualization

Below are the stiffness curves and associated events

GENERATED BY THE SECTIONPRO SOFTWARE

Figure 11: PDF export, rectangular section.



STIFFNESS CURVES CALCULATION RESULTS

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Events

Events correspond to special situations during the gradual construction of the stiffness curve where a material (concrete or steel) reaches an elastic or plastic deformation:

Param	Units	#1	#2	#3
Material		Steel	Concrete	Steel
εc / εs	‰	2.174	-2.000	10.000
χy	‰	0.742	2.547	3.119
My	kN·m	25324.0	37355.6	38006.3
EI (sec)	kN·m²	3.411 E7	1.466 E7	1.219 E7
EI (tan)	kN·m²	3.244 E7	1.439 E6	9.154 E5

Visualization

Below are the stiffness curves and associated events

GENERATED BY THE SECTIONPRO SOFTWARE

Figure 12: PDF export, oblong section.

Conclusion

The stiffness curve module provides the true evolution of flexural and axial stiffness as a function of the loading state. By sweeping a force component from zero to failure, it captures the full degradation path, from the initial elastic response through progressive yielding to rupture, and reports the curvatures and axial strains at each load level.

The secant and tangent stiffnesses (EI_{sec}, EI_{tan}, EA) give engineers the actual stiffness values to use in structural models, replacing the conventional assumption of constant EI. The automatically detected stiffness events identify the key transitions on the constitutive laws (elastic-to-plastic, plateau, and rupture) with their associated force levels and stiffness values.