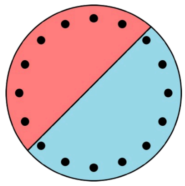

Steel Dimensioning from Influence Surfaces

SectionPro Tutorial: finding the minimum reinforcement
for a batch of load combinations using the 3D resistance domain

BridgeKernel · 2026

Introduction

SectionPro offers two verification workflows: the cross-section equilibrium solver (Article #3), which processes any number of load combinations and returns the detailed stress/strain state for each one individually; and the interaction-surface verification (Article #5), which builds the 3D resistance domain $\left(N, M_z, M_y\right)$ and evaluates all loads at once by measuring their normalized distance η to the surface. Both answer the verification question: is the section adequate for a given φ_s?

This article addresses the inverse problem, dimensioning: given a reinforcement layout and a set of load combinations, find the minimum bar diameter φ_s such that every load falls inside the interaction surface. The algorithm searches for the diameter that produces a surface where the most critical load arrives exactly at the boundary ($\eta = 1$), up to a numerical tolerance criterion. Each limit state is solved independently and SectionPro reports φ_s per state as well as the governing value across all states.

For ACI 318, CSA A23.3, and AASHTO, the surface is built natively from the Whitney stress block with strength reduction factors. Because loads are evaluated geometrically rather than by iterative convergence, this method becomes significantly faster for large load envelopes (see Section 5).

Computed results

SectionPro reports three categories of results:

Steel dimensioning

φ_s: required bar diameter
A_s: required steel area per bar
One result per limit state
Governing φ_s across all limit states

Distances & 3D view

η: normalized distance per load
Status: Internal / Boundary
3D scatter on final surface
One surface per limit state

Exports

PDF: 3D views + dimensioning table
XLS: φ_s, loads, distances, status
TXT: tabular results (columns)

Octagonal section (Eurocode 2)

Input data

The section geometry, reinforcement layout, and material laws are identical to those used in Articles #4 and #5. 30 load combinations are defined: 15 at ULS-F (Fundamental) and 15 at SLS-C (Characteristic).

Concrete

- Octagonal cross section
- $b_1 = 2.00$ m, $b_2 = 0.50$ m
- $h_1 = 1.00$ m, $h_2 = 0.60$ m

Reinforcement layout

- 48 bar positions, uniform spacing 150 mm
- Cover 50 mm, 1 layer
- Diameter φ_s: to be determined

Material laws (EC2)

- Concrete C30/37: $f_{ck} = 30$ MPa
- Steel B500B: $f_{yk} = 500$ MPa

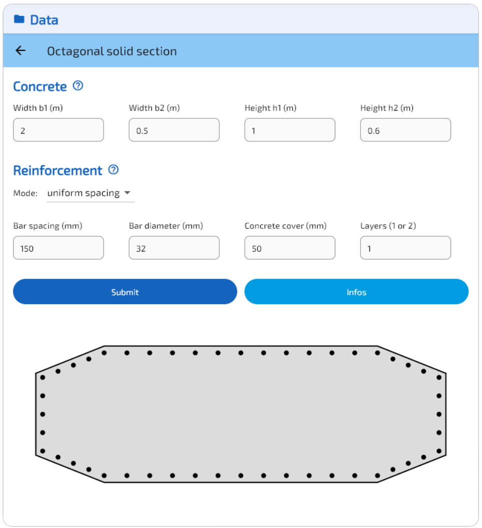

Figure 1: Octagonal section: geometry and reinforcement layout.

Dimensioning results

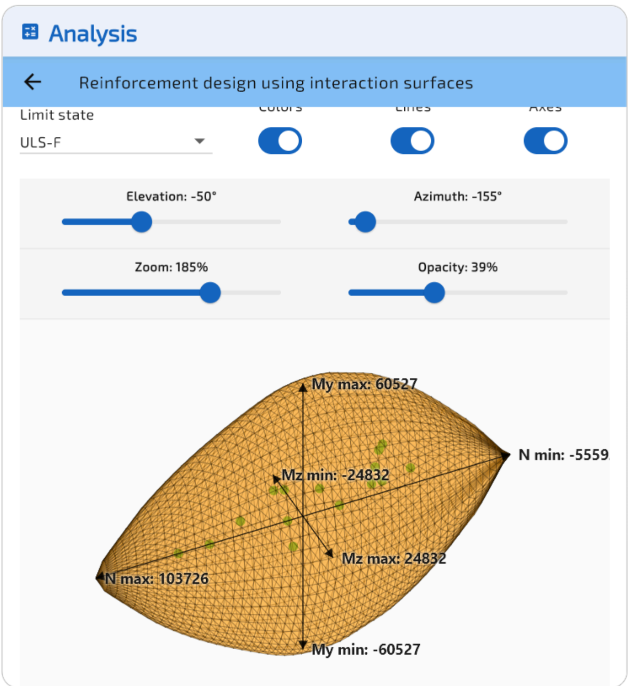

Figure 2: ULS-F: interaction surface at converged φ_s.

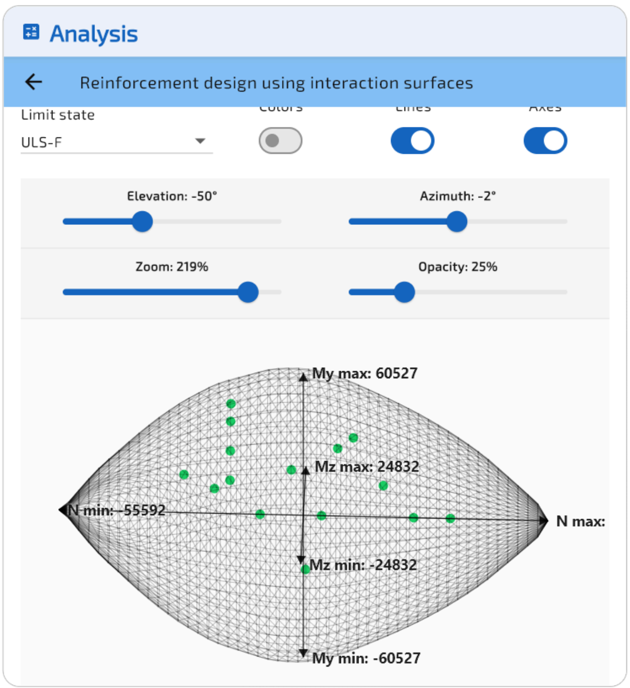

Figure 3: ULS-F, rotated.

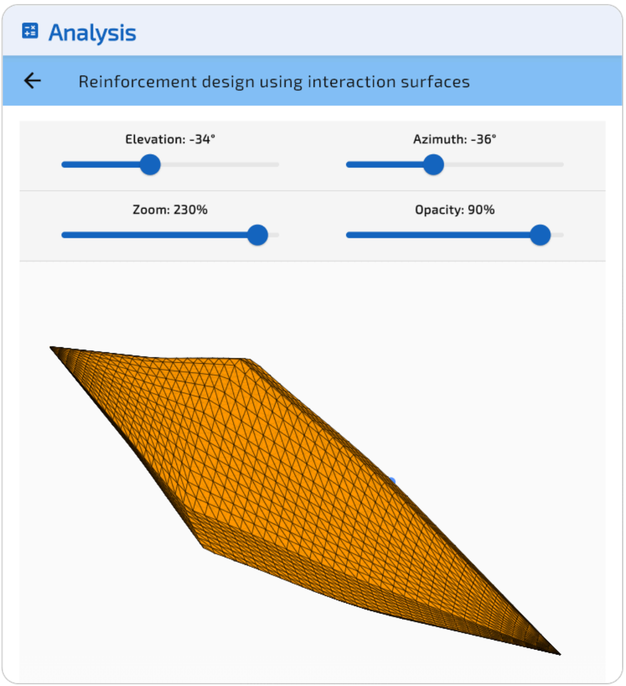

Figure 4: SLS-C: interaction surface at converged φ_s.

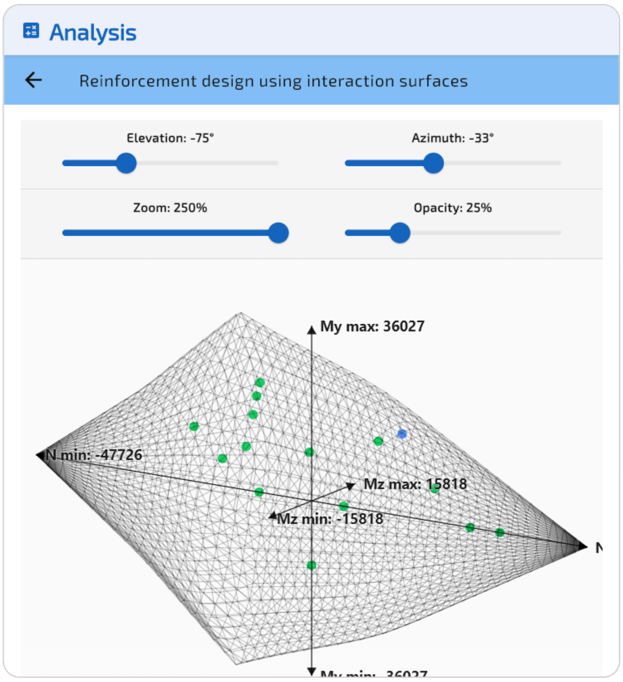

Figure 5: SLS-C, rotated.

SLS-C governs with $\varphi_s = 56.26$ mm (vs 55.09 mm at ULS-F). At the governing diameter, the SLS-C boundary load (#26) is visible in blue on the surface, while all ULS-F loads are internal (green).

State	φ_s (mm)	η_{worst}	Worst load	Status
ULS-F	55.09	0.970	#8	Internal
SLS-C	56.26	1.000	#26	Boundary

Distances at the governing reinforcement

Once the governing $\varphi_s = 56.26$ mm is determined (SLS-C controls), SectionPro rebuilds the interaction surface for each limit state at this diameter and computes distances for all 30 loads. Every load must be Internal ($\eta < 1$) or at the boundary ($\eta \approx 1$).

Load	State	N (kN)	M_z (kN·m)	M_y (kN·m)	η	Status
26	SLS-C	35000	6000	15000	1.000	Boundary
8	ULS-F	0	14000	35000	0.970	Internal
23	SLS-C	0	8000	20000	0.883	Internal
7	ULS-F	0	11000	30000	0.820	Internal
25	SLS-C	30000	5000	13000	0.815	Internal
22	SLS-C	0	6800	17500	0.797	Internal
28	SLS-C	−13000	4000	10000	0.796	Internal
11	ULS-F	40000	10000	25000	0.727	Internal
29	SLS-C	45000	2500	6000	0.716	Internal
4	ULS-F	72000	0	0	0.603	Internal

The remaining 20 loads all have $\eta < 0.60$. The full table can be exported by the software in PDF, TXT and XLS formats.

Elliptical section (ACI 318)

Input data

The section geometry, reinforcement, and material laws are identical to those used in the interaction-surface verification article. 30 load combinations are defined: 15 at ULS and 15 at SLS. The ACI 318 Whitney stress block is used natively to build the ULS interaction surface, including the strength reduction factors ($\varphi = 0.90$ tension-controlled, $\varphi = 0.65$ compression-controlled, $\varphi_N = 0.80$ cap). The SLS surface uses linear elastic behaviour with allowable stresses ($\sigma_c = 11.5$ MPa, $\sigma_s = 250$ MPa).

Concrete

- Elliptical cross section
- Width = 3.00 m, Height = 2.00 m

Reinforcement layout

- 40 bars along the perimeter
- Cover 50 mm
- Diameter φ_s: to be determined

Material laws (ACI 318)

- Concrete: $f'_c = 30$ MPa
- Steel: $f_y = 500$ MPa
- Whitney block: $\beta_1 = 0.832$

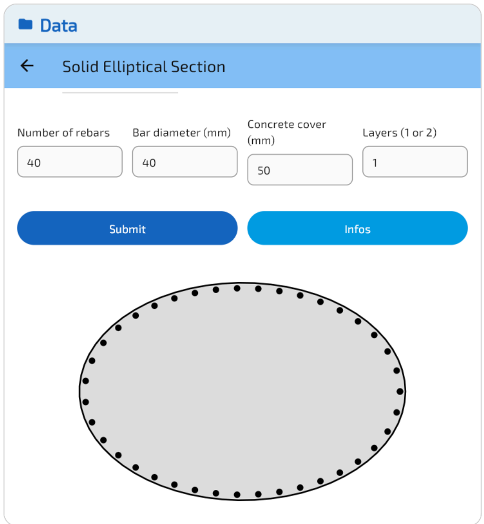

Figure 6: Elliptical section: geometry and reinforcement layout.

Dimensioning results

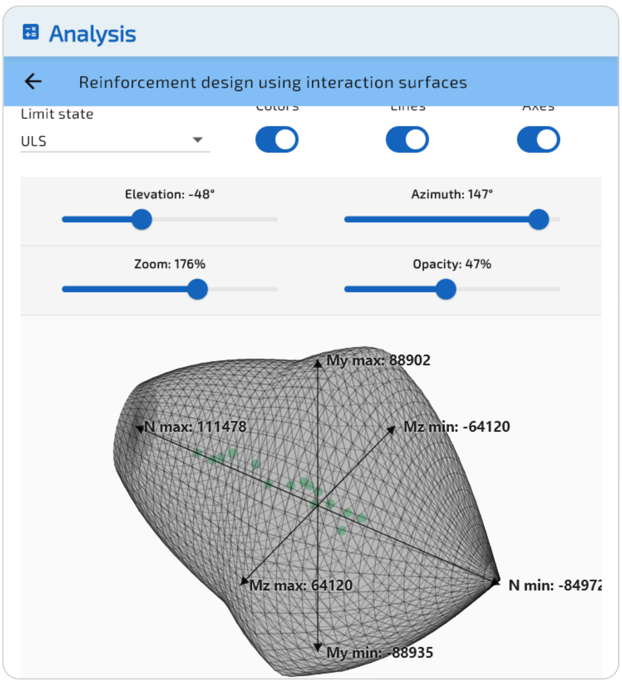

Figure 7: ULS: interaction surface at converged φ_s.

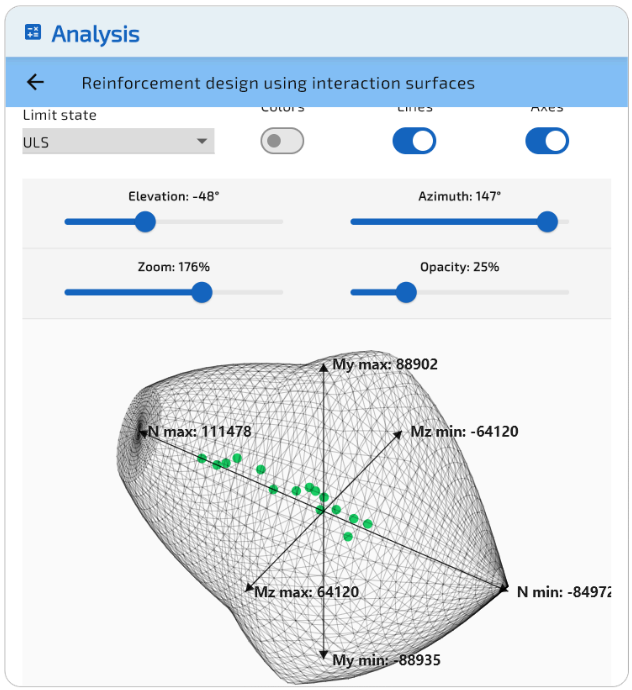

Figure 8: ULS, rotated.

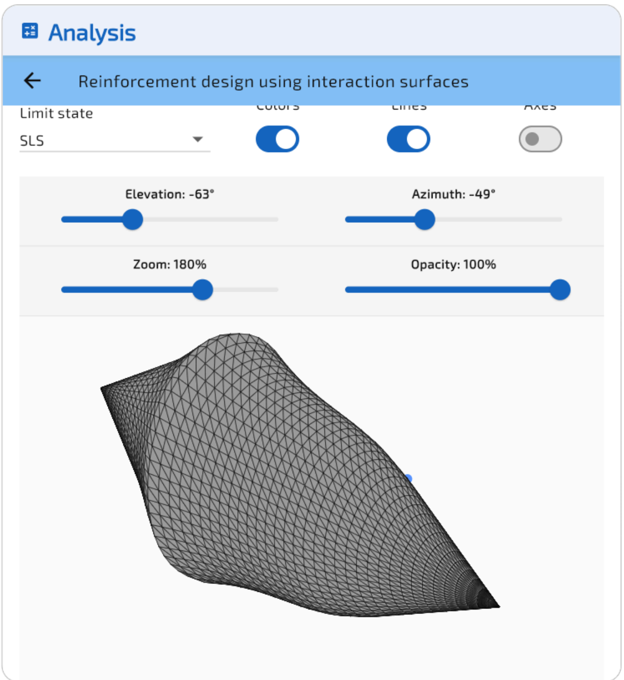

Figure 9: SLS: interaction surface at converged φ_s.

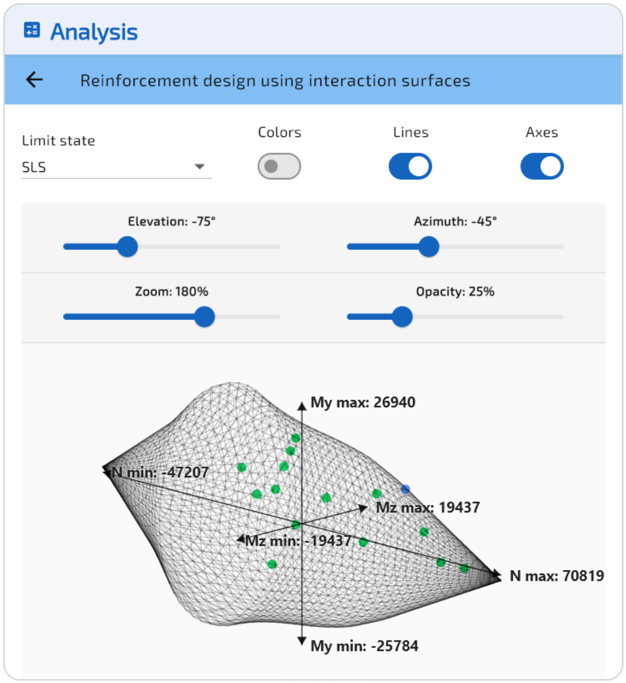

Figure 10: SLS, rotated.

SLS governs with $\varphi_s = 77.53$ mm (vs 64.71 mm at ULS). The SLS boundary load (#26) is visible in blue on the surface, while all ULS loads are internal (green).

State	φ_s (mm)	η_{worst}	Worst load	Status
ULS	64.71	0.781	#8	Internal
SLS	77.53	1.000	#26	Boundary

Distances at the governing reinforcement

At the governing $\varphi_s = 77.53$ mm (SLS controls), all 30 loads are Internal. The top 10 most critical loads are:

Load	State	N (kN)	M_z (kN·m)	M_y (kN·m)	η	Status
26	SLS	35000	7500	11000	1.000	Boundary
23	SLS	0	10000	15000	0.866	Internal
19	SLS	60000	0	0	0.835	Internal
29	SLS	45000	3000	4500	0.812	Internal
8	ULS	0	32000	45000	0.781	Internal
25	SLS	28000	6000	9000	0.766	Internal
18	SLS	53000	0	0	0.712	Internal
11	ULS	45000	22000	33000	0.710	Internal
7	ULS	0	27000	39000	0.699	Internal
22	SLS	0	8500	12500	0.695	Internal

Cross-validation with interaction-surface verification (Article #5)

The interaction-surface verification (Article #5) analysed the same two sections with fixed bar diameters ($\varphi = 32$ mm for the octagon, $\varphi = 40$ mm for the ellipse). At those diameters, several loads were classified as External ($\eta > 1$), meaning the section capacity was exceeded. The dimensioning module (this article) must therefore return φ_s values larger than those fixed diameters.

Octagonal section (EC2, fixed $\varphi = 32$ mm in Art. #5)

With $\varphi = 32$ mm, 7 of the 15 ULS-F loads were External and 8 of the 15 SLS-C loads were External. The dimensioning module returns $\varphi_s = 55.09$ mm (ULS-F) and $\varphi_s = 56.26$ mm (SLS-C), both well above 32 mm, confirming that the fixed diameter was insufficient. The governing SLS-C diameter is 76% larger than the verification diameter.

Limit state	φ Art.#5 (mm)	φ_s dim. (mm)	External in Art.#5
ULS-F	32	55.09	7 / 15
SLS-C	32	56.26	8 / 15

Elliptical section (ACI 318, fixed $\varphi = 40$ mm in Art. #5)

Similarly, with $\varphi = 40$ mm, 7 of the 15 ULS loads and 8 of the 15 SLS loads were External. The dimensioning module returns $\varphi_s = 64.71$ mm (ULS) and $\varphi_s = 77.53$ mm (SLS). The governing SLS diameter is 94% larger than the verification diameter.

Limit state	φ Art.#5 (mm)	φ_s dim. (mm)	External in Art.#5
ULS	40	64.71	7 / 15
SLS	40	77.53	8 / 15

In both cases, the dimensioning module correctly returns diameters that exceed the verification diameter whenever external loads were present, confirming full consistency between the verification and dimensioning workflows.

Cross-validation with cross-section equilibrium solver (Article #3)

The cross-section equilibrium solver (Article #3) computed the required φ_s for individual load cases on a solid hexagonal section (EC2, C30/37, 30 bars, 150 mm spacing). Two load cases at ULS-F were analysed: a combined biaxial loading and a uniaxial bending case. Running the dimensioning module on the same section with these two loads as the envelope, the governing φ_s must match the largest diameter found by the direct solver.

Hexagonal section — two-load envelope

Load	N (kN)	M_z (kN·m)	M_y (kN·m)	φ_s^{NR} (mm)	φ_s^{SI} (mm)
ULS	2000	3000	1800	10.73	—
SLS	500	1000	0	6.40	—
Gov.				10.73	10.82

Both solvers converge to the same governing diameter: $\varphi_s = 10.73$ mm (equilibrium) vs $\varphi_s = 10.82$ mm (interaction surface).

Performance benchmark

The following table compares the computation time on the octagonal EC2 section (5 limit states, 48 bars), scaling the number of load combinations from 2 to 1 000 000. Loads are distributed randomly across all limit states. The equilibrium solver performs one iterative convergence per load. The interaction-surface method builds the surface and evaluates all loads geometrically.

Loads	Direct solver (ms)	SI dimensioning (ms)	Speedup
2	2.1	154	0.01 ×
100	5.6	382	0.01 ×
1000	23	441	0.05 ×
10000	167	526	0.32 ×
100000	1688	1306	1.29 ×

Both methods deliver similar performance across all scales, with the equilibrium solver faster at small envelopes and the interaction-surface method catching up around 100000 loads. The φ_s values agree to within 0.1% at all scales. The practical advantage of the interaction-surface approach is not raw speed but the visual output: the 3D scatter plot on the governing surface gives immediate confirmation that all loads are enclosed, which the equilibrium solver does not provide.

Conclusion

The interaction-surface dimensioning provides an efficient way to determine the minimum reinforcement diameter for an entire load envelope:

- One run, all loads: the algorithm processes any number of load combinations in a single pass.
- Whitney-native for US codes: for ACI 318, CSA A23.3, and AASHTO, the surface is built directly from the Whitney stress block with strength reduction factors.
- Cross-validated: the dimensioning results match the interaction-surface verification (Article #5: loads that were External at the fixed diameter now require a larger φ_s) and the cross-section equilibrium solver (Article #3: both methods converge to the same governing diameter within numerical tolerance).
- Visual confirmation: the 3D scatter plot at the governing φ_s immediately shows that all loads are enclosed, with the critical load at the boundary.
- Complementary to the equilibrium solver: the equilibrium solver returns the full stress/strain state, while the interaction-surface method provides a visual envelope check at similar computational cost.

Export

SectionPro exports the dimensioning results in PDF, TXT and XLS formats. The PDF report includes 3D views of the final interaction surface with scattered load points and a results table.

Reinforcement design based on IS

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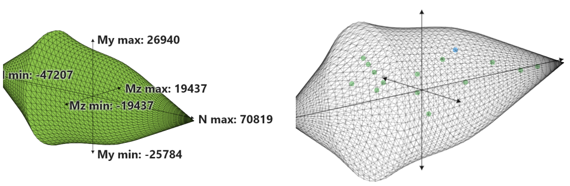
Minimum reinforcement diameters ϕs for each limit state:

State	SLS	ULS
ϕs (mm)	77.53	64.71
Load	#26	#8

The calculation performed allows dimensioning the reinforcement diameters such that the reinforced concrete section strictly satisfies the resistance requirements with respect to the given loads.

SLS : Serviceability limit state (ϕs = 77.53 mm)

14 loads are internal to the N-Mz-My surface. Scattered in green
0 loads are external to the N-Mz-My surface. Scattered in red
1 load is on the boundary of the N-Mz-My surface. Scattered in blue



ULS : Ultimate limit state (ϕs = 64.71 mm)

15 loads are internal to the N-Mz-My surface. Scattered in green
0 loads are external to the N-Mz-My surface. Scattered in red
0 loads are on the boundary of the N-Mz-My surface. Scattered in blue

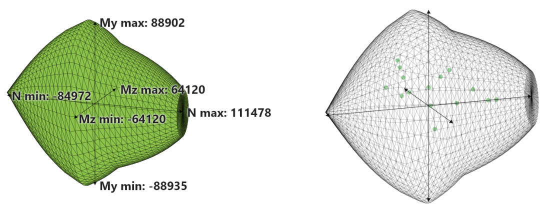


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Figure 11: PDF export page 1: 3D views.

Distances of loads to surface

Load case #26 is the most unfavorable
Distances are calculated considering the largest calculated ϕs.

Load	Limit state	N (kN)	Mz (kN·m)	My (kN·m)	SF	Status
26	SLS	35000.0	7500.0	11000.0	1.000	Boundary
23	SLS	0.0	10000.0	15000.0	0.866	Internal
19	SLS	60000.0	0.0	0.0	0.835	Internal
29	SLS	45000.0	3000.0	4500.0	0.812	Internal
8	ULS	0.0	32000.0	45000.0	0.781	Internal
25	SLS	28000.0	6000.0	9000.0	0.766	Internal
18	SLS	53000.0	0.0	0.0	0.712	Internal
11	ULS	45000.0	22000.0	33000.0	0.710	Internal
7	ULS	0.0	27000.0	39000.0	0.699	Internal
22	SLS	0.0	8500.0	12500.0	0.695	Internal
13	ULS	-20000.0	15000.0	20000.0	0.647	Internal
28	SLS	-11000.0	5000.0	7500.0	0.620	Internal
6	ULS	0.0	20000.0	30000.0	0.587	Internal
4	ULS	78000.0	0.0	0.0	0.539	Internal
10	ULS	35000.0	18000.0	28000.0	0.521	Internal
21	SLS	0.0	6500.0	9500.0	0.492	Internal
30	SLS	8000.0	-5000.0	-8000.0	0.490	Internal
15	ULS	10000.0	-15000.0	-25000.0	0.472	Internal
12	ULS	-8000.0	8000.0	12000.0	0.470	Internal
5	ULS	0.0	10000.0	15000.0	0.434	Internal
3	ULS	70000.0	0.0	0.0	0.429	Internal
24	SLS	15000.0	4000.0	6000.0	0.407	Internal
14	ULS	60000.0	8000.0	12000.0	0.398	Internal
27	SLS	-4000.0	2500.0	3500.0	0.375	Internal
20	SLS	0.0	4000.0	5000.0	0.367	Internal

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Figure 12: PDF export page 2: results table.