

Second-Order Effects

SectionPro Tutorial — Buckling analysis of circular columns
with the EC2 nominal curvature method (uniaxial and biaxial)

BridgeKernel · 2026

Introduction

A second-order analysis estimates the amplification of bending moments caused by the compression force acting on the deformed shape of the column. The initial eccentricity e_0 triggers a lateral deflection, which in turn increases the bending moment, which increases the deflection further. The Eurocode's nominal curvature method quantifies this effect through a second-order eccentricity e_2 derived from the section curvature $1/r$ and the effective buckling length l_0 , without requiring a full nonlinear FEM model:

$$e_2 = \frac{1}{r} \cdot \frac{l_0^2}{\pi^2} \quad M_{Ed} = N \cdot (e_0 + e_2) + M_1$$

SectionPro evaluates the curvature $1/r$ from a nonlinear analysis at each load level, then applies the Eurocode formula above to obtain e_2 . The axial force N is increased, tracing a load path on the interaction diagram until either the material capacity is reached (resistance) or the eccentricity diverges (buckling instability). Two modes are available:

- 2D uniaxial: buckling is analysed in one bending plane (N - M_z or N - M_y). The other moment component is held constant, and an optional first-order moment M_1 can be imposed.
- 3D biaxial: both bending planes are amplified simultaneously, each with its own buckling length $l_{k,x}$ and $l_{k,y}$. The load path is traced on the full 3D interaction surface.

Computed results

SectionPro reports for each buckling analysis:

Load path

Axial compressive force N increased until failure
Eccentricities $e_{2,x}$, $e_{2,y}$ at each load level
Total moment M_{Ed} including second-order effects

Capacity reduction

Ratio of failure point to curve intersection
Based on material strain limits
Status: resistance or buckling
 $e_{2,max}$ at failure point

Exports

PDF: load path plotted on the interaction curve/surface
XLS and TXT: full path data (N , M_z , M_y , $e_{2,x}$, $e_{2,y}$)

Solid circular column (slender)

Input data

Concrete

- Solid circular cross-section
- Diameter $D = 1.00$ m
- Area $A_c = 0.785$ m²

Reinforcement

- 20 bars HA25 ($\varphi_s = 25$ mm)
- Positioned at $r = 447.5$ mm
- Cover 40 mm — 1 layer
- $A_s = 20 \times 4.91 = 98.2$ cm²

Material laws (EC2)

- Concrete C30/37: $f_{ck} = 30$ MPa
- Steel B500B: $f_{yk} = 500$ MPa

The screenshot shows a software interface for defining a solid circular column section. The 'Concrete' section has a diameter of 1 m. The 'Reinforcement' section is set to 'count' mode with 20 bars, 25 mm diameter, 40 mm cover, and 1 layer. A circular diagram below shows the reinforcement layout.

Figure 1: Solid circular column.

2D uniaxial buckling ($N-M_z$ plane)

The column has an effective buckling length of $l_k = 30.0$ m with an initial eccentricity $e_0 = 0.10$ m and no first-order moment ($M_1 = 0$).

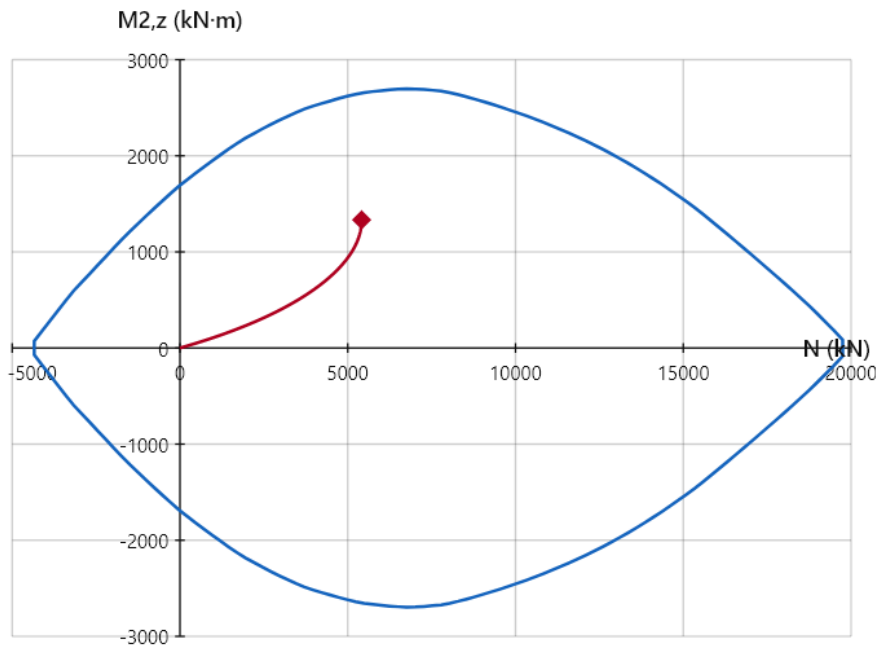


Figure 2: Load path on the $N-M_z$ interaction curve — strong curvature indicates significant second-order effects.

With a slenderness ratio $\lambda = l_k/i = 120$, this column is highly slender. The load path is nearly linear up to about $N = 3500$ kN, where second-order effects remain small. Beyond this point, the eccentricity e_2 grows rapidly and the load path curves sharply upward. The column fails by geometric instability at:

- $N_{crit} = 5318$ kN

- Capacity reduction: 53%
- At 25% of the maximum compressive resistance ($N = 5000 \text{ kN}$ out of 20000 kN): total moment $M_{Ed} = 928 \text{ kN} \cdot \text{m}$, of which $M_2 = 430 \text{ kN} \cdot \text{m}$ is second-order (46%)

For this slender column, second-order effects are already severe at a fraction of the axial capacity.

Hollow circular column

Input data

Concrete

- Hollow circular cross-section
- Outer diameter $D_{ext} = 2.50 \text{ m}$
- Wall thickness $t = 0.35 \text{ m}$
- Inner diameter $D_{int} = 1.80 \text{ m}$

Reinforcement

- 30 bars HA20 ($\varphi_s = 20 \text{ mm}$)
- Positioned at $r = 1200 \text{ mm}$ (outer layer)
- Cover 40 mm — 1 layer
- $A_s = 30 \times 3.14 = 94.2 \text{ cm}^2$

Material laws (EC2)

- Concrete C30/37: $f_{ck} = 30 \text{ MPa}$
- Steel B500B: $f_{yk} = 500 \text{ MPa}$

The screenshot shows a 'Data' window for a 'Hollow Circular Section'. Under 'Concrete', the 'Outer diameter (m)' is 2.5 and 'Wall thickness (m)' is 0.35. Under 'Reinforcement', the 'Mode' is 'count', 'Number of rebars' is 30, 'Bar diameter (mm)' is 20, 'Concrete cover (mm)' is 40, and 'Layers (1 or 2)' is 1. There are 'Submit' and 'Infos' buttons. Below the form is a diagram of a hollow circular column with reinforcement bars.

Figure 3: Hollow circular column.

2D uniaxial buckling ($N-M_z$ plane)

The column has an effective buckling length of $l_k = 30.0 \text{ m}$ with an initial eccentricity $e_0 = 0.10 \text{ m}$ and no first-order moment ($M_1 = 0$).

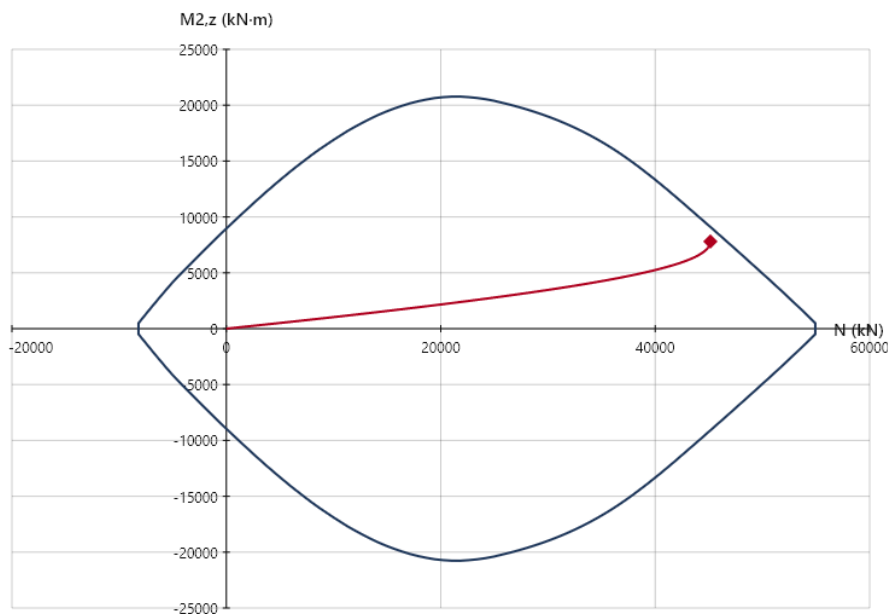


Figure 4: Load path on the $N-M_z$ interaction curve — nearly linear, second-order effects are small.

With a slenderness ratio $\lambda = l_k/i = 39$, this column is stocky. The load path is nearly linear for most of the range, but the eccentricity e_2 starts to accelerate noticeably beyond $N = 35000$ kN. Unlike the slender column where this acceleration occurs early, here it only appears when N is already close to the maximum compressive resistance. The column fails just before reaching the interaction curve:

- $N_{\text{crit}} = 42154$ kN
- Capacity reduction: 1.3%
- At 25% of the maximum compressive resistance ($N = 13000$ kN out of 51000 kN): total moment $M_{\text{Ed}} = 1560$ kN · m, of which $M_2 = 260$ kN · m is second-order (17%)

Second-order effects only become noticeable as N approaches the maximum compressive resistance.

3D biaxial buckling

In 3D mode, SectionPro amplifies the bending moments in both planes simultaneously. Each direction has its own buckling length ($l_{k,x}$, $l_{k,y}$) and initial eccentricity ($e_{0,x}$, $e_{0,y}$), and the second-order eccentricities $e_{2,x}$ and $e_{2,y}$ are computed independently at each load level.

The hollow circular column is analysed with symmetric buckling lengths: $l_{k,z} = l_{k,y} = 10.0$ m with $e_{0,z} = e_{0,y} = 0.05$ m. No first-order moments are applied.

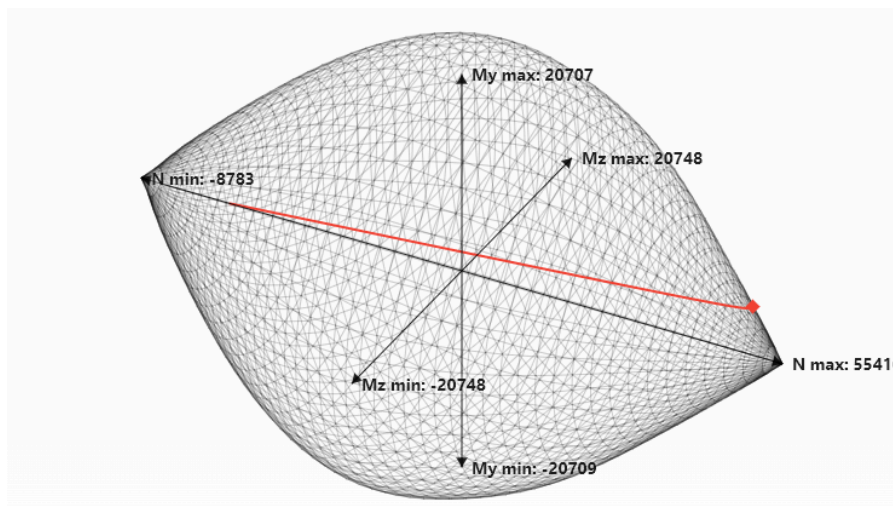


Figure 5: 3D load path on the interaction surface.

With a short buckling length of 10 m, second-order effects are negligible throughout the load path. The eccentricity e_2 remains below 1 mm for most of the range and only reaches $e_2 = 7.8$ mm at the very last point. The load path is essentially linear and reaches the interaction surface at:

- $N_{\text{crit}} = 50903$ kN
- At 25% of the maximum compressive resistance ($N = 12500$ kN out of 51000 kN): total moment $M_{\text{Ed}} = 626$ kN · m per axis, of which $M_2 = 3$ kN · m is second-order (0.5%)

The column reaches its full material resistance with virtually no capacity reduction from geometric effects, mainly due to the reduced buckling lengths and smaller initial eccentricities compared to the 2D examples.

Performance benchmark

The second-order analysis consists of two phases: building the interaction curve (or surface), then tracing the load path by incrementally computing e_2 at each load level. Each step evaluates the section curvature through an iterative algorithm. The table below shows the total computation time for 500 load path points.

Solid circular (2D)	Hollow circular (2D)	Hollow circular (3D)
91 ms	252 ms	282 ms

The dominant cost is building the interaction surface. The load path tracing itself adds only a few milliseconds, keeping the total analysis well under 300 ms in all cases.

Export

SectionPro exports the buckling analysis in three formats: PDF, text, and Excel (.xlsx). The exported data includes the full load path (N , M_z , M_y , $e_{2,x}$, $e_{2,y}$ at each load level), the capacity reduction factor, and the buckling status.

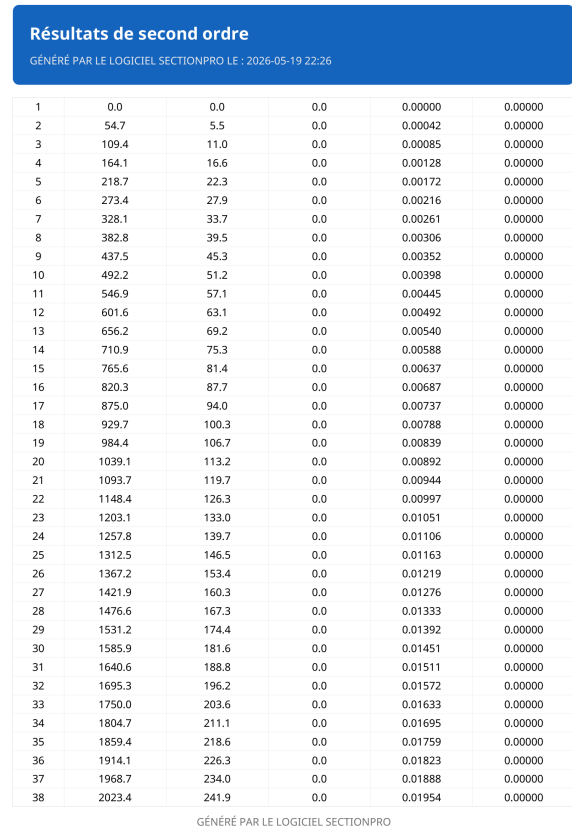
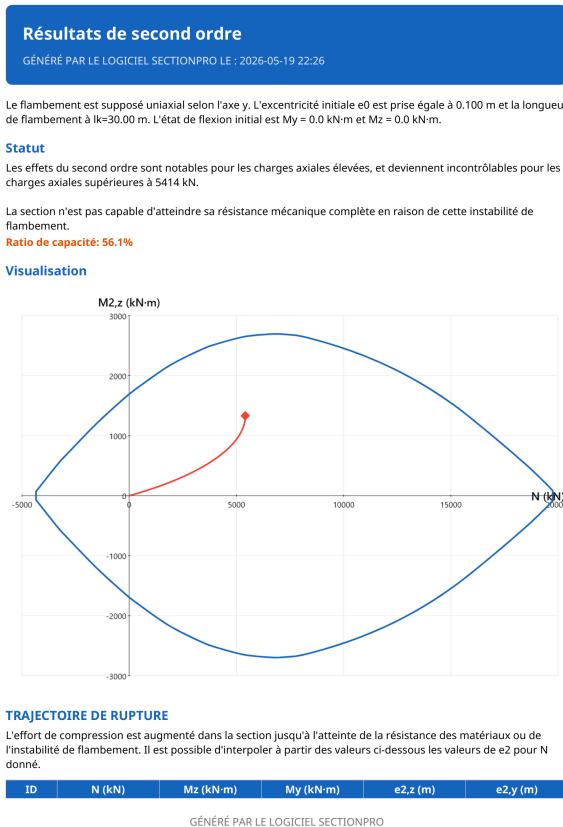


Figure 6: PDF export — page 1: load path on interaction curve.

Figure 7: PDF export — page 2: detailed results table.

| Conclusion

The nominal curvature method allows engineers to evaluate second-order effects at the section level without the cost and complexity of a full nonlinear FEM model. The load path visualisation on the interaction curve (or surface) provides an immediate assessment of how significant second-order effects are for a given column.

The comparison between the solid $\text{Ø}1\text{m}$ column and the hollow $\text{Ø}2.5\text{m}$ column demonstrates that geometric properties, not just the buckling length, govern the outcome. The slender solid column exhibits a strongly curved load path and fails by instability, while the hollow column reaches its material capacity with small second-order amplification.

The 3D biaxial mode extends this analysis to columns with different buckling lengths in each direction, amplifying moments independently in both planes.