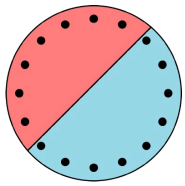


Reinforcement Design

SectionPro Tutorial — Required reinforcement for hexagonal, hollow square & U-beam sections under SLS and ULS loads (EC2, NBR-6118, BAEL 91)

BridgeKernel · 2026

Introduction

Given a set of imposed internal forces (N, M_y, M_z) and a predefined reinforcement layout (bar positions and spacing), SectionPro determines the minimum bar diameter φ_s required to satisfy the normative limits at each bar location. This is the inverse problem of the stress verification analysis (Article #2): instead of checking whether a given reinforcement is sufficient, the software finds the reinforcement that achieves equilibrium under the imposed loads.

The solver iterates on φ_s until the strain state $(\varepsilon_0, \kappa_y, \kappa_z)$ satisfies internal equilibrium with the normative strain limits exactly reached. When the concrete alone can resist the imposed loads without reinforcement, the result is $A_s = 0$ — no steel is needed.

This article uses the same three sections and the same load cases as Article #2. In Article #2, the reinforcement was fixed and some load cases exceeded the section capacity (FS > 1, check KO). Here, we determine the reinforcement that would be needed. The correlation is direct: a higher FS in Article #2 means a larger φ_s is required in Article #3.

Computed results

SectionPro reports three categories of results for each load case:

Stresses & strains + design

σ_c — Extreme concrete stress
$\sigma_{s,\min}, \sigma_{s,\max}$ — Steel stresses
ε_c — Extreme concrete strain
$\varepsilon_{s,\min}, \varepsilon_{s,\max}$ — Steel strains
Pivot — Failure mode (A, B, or $A_s = 0$)
φ_s — Required bar diameter

Internal forces

N_c — Compression resultant
N_t — Tension resultant
(x_C, y_C) — Compression centroid
(x_T, y_T) — Tension centroid
z — Internal lever arm

Convergence

N_{iter} — Iterations
Tol — Convergence tolerance
$N_{\text{int}}, M_{z,\text{int}}, M_{y,\text{int}}$ — Internal forces
$\varepsilon_0, \kappa_x, \kappa_y$ — Strain state

Failure pivots

The failure pivot indicates which material reaches its ultimate strain first:

- **Pivot A** — **Steel failure.** The tensile reinforcement reaches its ultimate strain ε_{su} before the concrete crushes. Typical of lightly reinforced or tension-dominated sections. Governing strain: $\varepsilon_s = \varepsilon_{su}$.
- **Pivot B** — **Concrete failure.** The concrete reaches its ultimate compressive strain ε_{cu} before the steel yields fully. Typical of heavily loaded or compression-dominated sections. Governing strain: $\varepsilon_c = \varepsilon_{cu}$.
- **Pivot C** — **Heavy compression.** The section is heavily compressed. The strain reaches $\varepsilon_c = \varepsilon_{c2}$ at a specific point located at $(1 - \varepsilon_{c2}/\varepsilon_{cu2}) \cdot h$ from the most compressed fibre (i.e. $3h/7$ for the common values $\varepsilon_{c2} = 2‰$ and $\varepsilon_{cu2} = 3.5‰$). Rare scenario in practice.
- **Pivot $A_s = 0$** — **No reinforcement needed.** The concrete alone can resist the imposed loads. The required steel area is zero.

Solid hexagonal section

Input data

Concrete

- Hexagonal cross section
- Width $B = 2.00$ m
- Minimum thickness $h_1 = 0.60$ m
- Maximum thickness $h_2 = 1.00$ m

Reinforcement layout

- Uniform spacing 150 mm
- 30 bar positions
- Cover 50 mm — 1 layer
- Diameter φ_s: **to be determined**

Material laws (EC2)

- Concrete C30/37: $f_{ck} = 30$ MPa
- Steel B500B: $f_{yk} = 500$ MPa

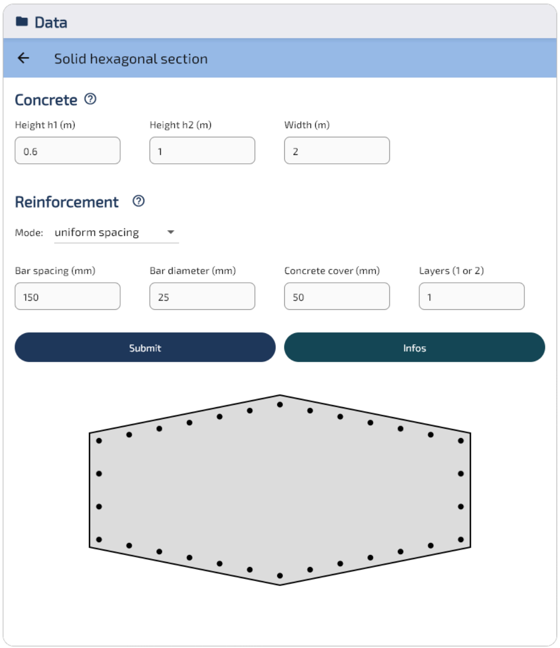


Figure 1: Hexagonal section.

SLS — Combined bending $(N + M_z)$

Imposed loads: $N = 500$ kN, $M_z = 1000$ kN · m, $M_y = 0$

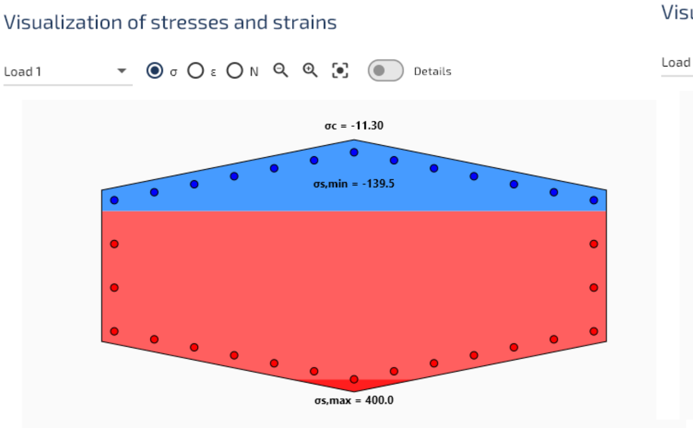


Figure 2: Stress distribution.

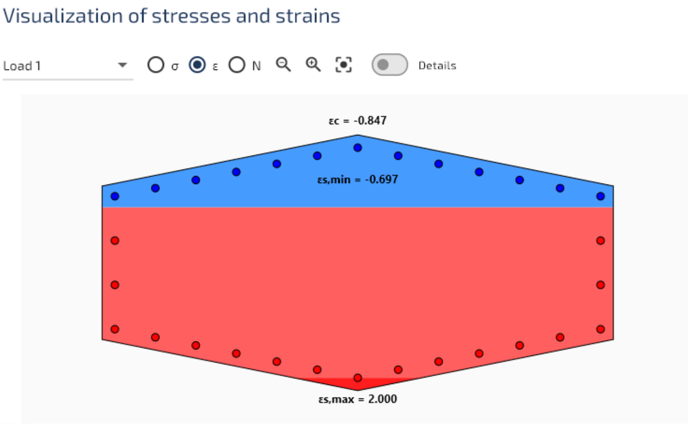


Figure 3: Strain distribution.

Stresses & strains + design

σ_c	-11.30 MPa
$\sigma_{s,\min}$	-139.49 MPa
$\sigma_{s,\max}$	400.00 MPa
ε_c	$-0.847‰$
$\varepsilon_{s,\min}$	$-0.697‰$
$\varepsilon_{s,\max}$	$2.000‰$
Pivot	A
φ_s	**17.60 mm**

Internal forces

N_c	1697.8 kN
N_t	-1197.8 kN
x_C	0.000 m
y_C	0.364 m
x_T	0.000 m
y_T	-0.320 m
z	0.683 m

Convergence

N_{iter}	4
Tol	3.57×10^{-8}
N_{int}	500.0 kN
$M_{z,\text{int}}$	1000.0 kN · m
$M_{y,\text{int}}$	0.0 kN · m
ε_0	0.651×10^{-3}
κ_x	-2.997×10^{-3}
κ_y	0.000×10^{-3}

Pivot A: the steel governs ($\varepsilon_{s,\max} = 2.000‰ = \varepsilon_{su}$). The required diameter is $\varphi_s = 17.60$ mm for all 30 bar positions.

ULS — Biaxial bending ($N + M_y + M_z$)

Imposed loads: $N = 2000$ kN, $M_z = 3000$ kN · m, $M_y = 1800$ kN · m

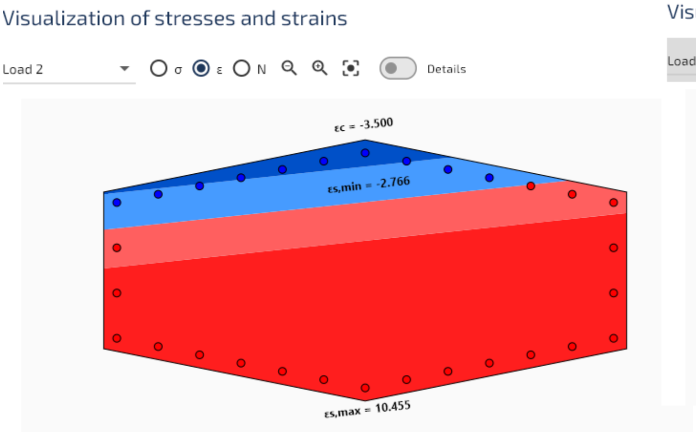


Figure 4: Stress distribution.

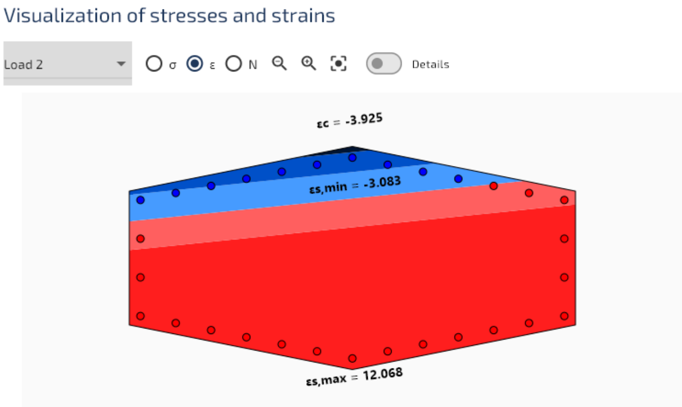


Figure 5: Strain distribution.

Stresses & strains + design

σ_c	-20.00 MPa
$\sigma_{s,\min}$	-435.21 MPa
$\sigma_{s,\max}$	440.81 MPa
ε_c	$-3.500‰$
$\varepsilon_{s,\min}$	$-2.766‰$
$\varepsilon_{s,\max}$	$10.455‰$
Pivot	B
φ_s	**25.12 mm**

Internal forces

N_c	5827.2 kN
N_t	-3827.2 kN
x_C	-0.255 m
y_C	0.355 m
x_T	0.082 m
y_T	-0.243 m
z	0.687 m

Convergence

N_{iter}	43
Tol	3.66×10^{-8}
N_{int}	2000.0 kN
$M_{z,\text{int}}$	3000.0 kN · m
$M_{y,\text{int}}$	1800.0 kN · m
ε_0	3.845×10^{-3}
κ_x	-14.689×10^{-3}
κ_y	-1.556×10^{-3}

Pivot B: the concrete governs ($\varepsilon_c = -3.500‰ = \varepsilon_{cu}$). The required diameter is $\varphi_s = 25.12$ mm for the ULS biaxial loading.

Hollow square section

Input data

Concrete

- Hollow square section
- Outer side $a = 2.0$ m
- Wall thickness $t = 0.30$ m

Reinforcement layout

- Uniform spacing 150 mm
- 64 bar positions
- Cover 40 mm
- 1 layer per face (inner + outer)
- Diameter φ_s: **to be determined**

Material laws (NBR-6118)

- Concrete C30: $f_{ck} = 30$ MPa
- Steel: $f_{yk} = 500$ MPa

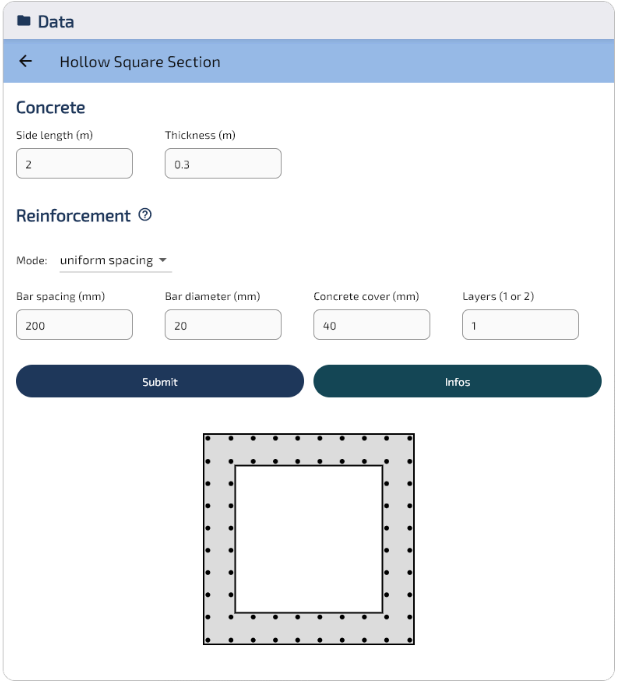


Figure 6: Hollow square section — geometry and reinforcement layout.

SLS — Biaxial bending $(N + M_y + M_z)$

Imposed loads: $N = -400$ kN, $M_z = 900$ kN · m, $M_y = 400$ kN · m

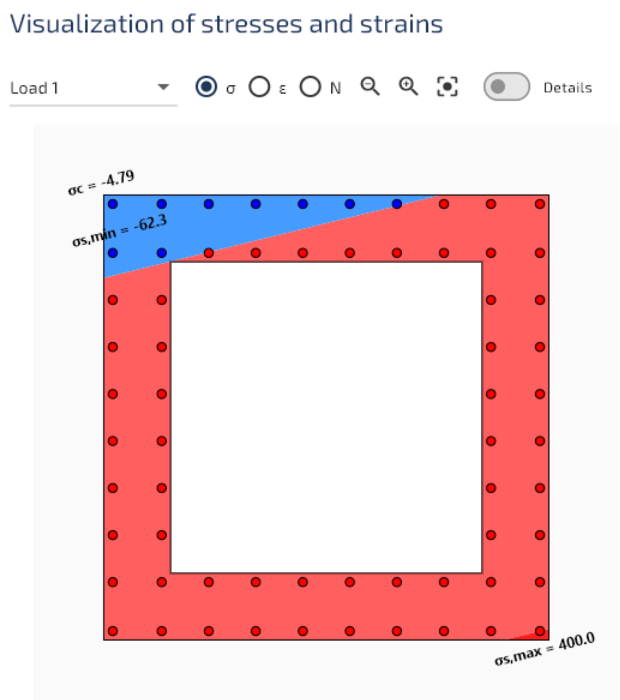


Figure 7: Stress distribution.

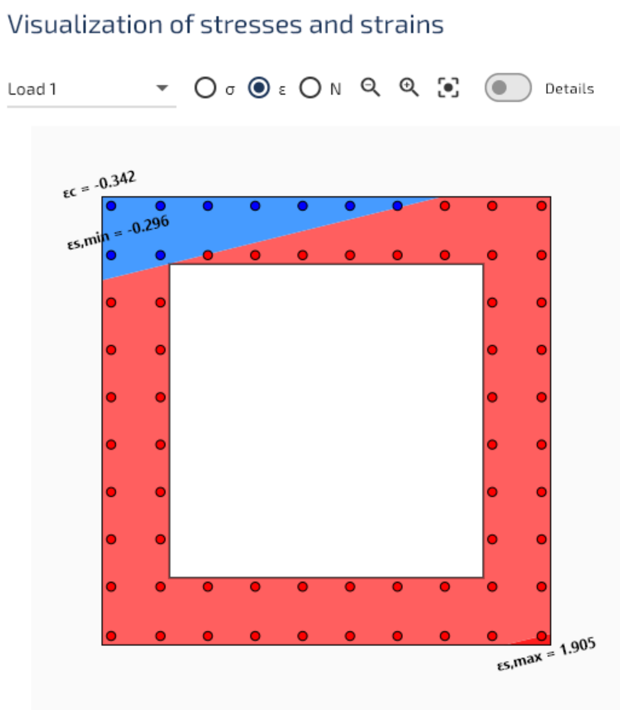


Figure 8: Strain distribution.

Stresses & strains + design

σ_c	-4.79 MPa
$\sigma_{s,\min}$	-62.26 MPa
$\sigma_{s,\max}$	400.02 MPa
ε_c	$-0.342‰$
$\varepsilon_{s,\min}$	$-0.296‰$
$\varepsilon_{s,\max}$	$1.905‰$
Pivot	A
φ_s	**10.00 mm**

Internal forces

N_c	468.2 kN
N_t	-868.2 kN
x_C	-0.622 m
y_C	0.908 m
x_T	0.126 m
y_T	-0.547 m
z	1.636 m

Convergence

N_{iter}	575
Tol	2.63×10^{-8}
N_{int}	-400.0 kN
$M_{z,\text{int}}$	900.0 kN · m
$M_{y,\text{int}}$	400.0 kN · m
ε_0	0.804×10^{-3}
κ_x	-0.920×10^{-3}
κ_y	-0.226×10^{-3}

Pivot A: the steel governs ($\varepsilon_{s,\max} = 2.000‰ = \varepsilon_{su}$). The required diameter is $\varphi_s = 10.00$ mm.

ULS — Biaxial bending ($N + M_y + M_z$)

Imposed loads: $N = 0$ kN, $M_z = 6000$ kN · m, $M_y = 6000$ kN · m

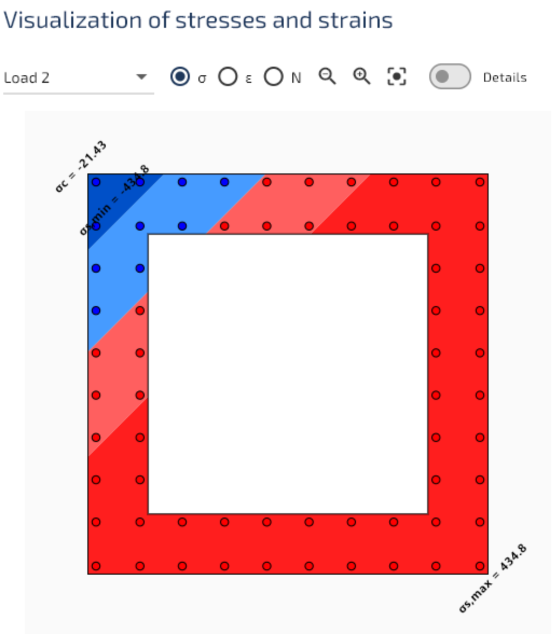


Figure 9: Stress distribution.

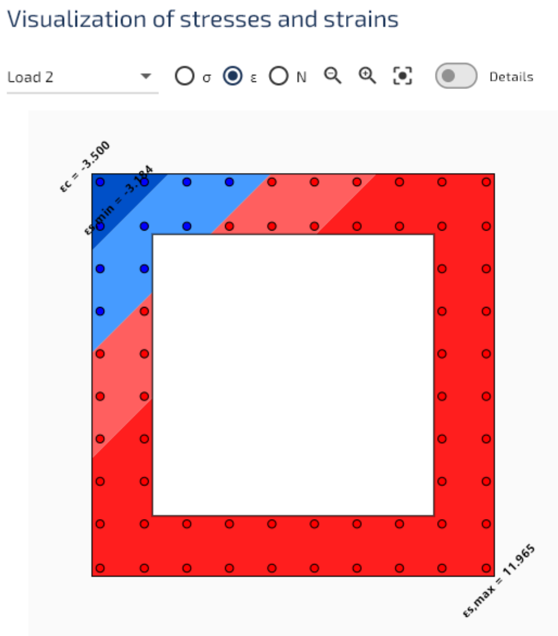


Figure 10: Strain distribution.

Stresses & strains + design

σ_c	-21.43 MPa
$\sigma_{s,\min}$	-434.78 MPa
$\sigma_{s,\max}$	434.78 MPa
ε_c	$-3.500‰$
$\varepsilon_{s,\min}$	$-3.184‰$
$\varepsilon_{s,\max}$	$11.965‰$
Pivot	B
φ_s	**19.38 mm**

Internal forces

N_c	5901.2 kN
N_t	-899.0 kN
x_C	-0.756 m
y_C	0.751 m
x_T	-0.179 m
y_T	0.421 m
z	0.664 m

Convergence

N_{iter}	41
Tol	3.40×10^{-8}
N_{int}	0.0 kN
$M_{z,\text{int}}$	6000.0 kN · m
$M_{y,\text{int}}$	6000.0 kN · m
ε_0	4.390×10^{-3}
κ_x	-3.945×10^{-3}
κ_y	-3.945×10^{-3}

Pivot B: the concrete governs ($\varepsilon_c = -3.500‰ = \varepsilon_{cu}$). The required diameter is $\varphi_s = 19.38$ mm for the ULS biaxial loading.

Custom section — U-beam

Input data

This section uses the **custom solid geometry** feature. The external contour is defined as a list of XY points, and the reinforcement layout is provided as a table of (x, y) positions. This is the recommended procedure for non-standard geometries that do not fit predefined parametric shapes.

Concrete

- U-beam with inclined webs
- Total height $h = 1.20$ m

Reinforcement layout

- Uniform spacing 150 mm
- Bottom slab: 11 positions
- Webs: 49 positions
- 2 layers per web
- Diameter φ_s: **to be determined**

Material laws (BAEL 91)

- Concrete: $f_{c28} = 30$ MPa, $\theta = 0.85$
- Steel fe500: $f_e = 500$ MPa

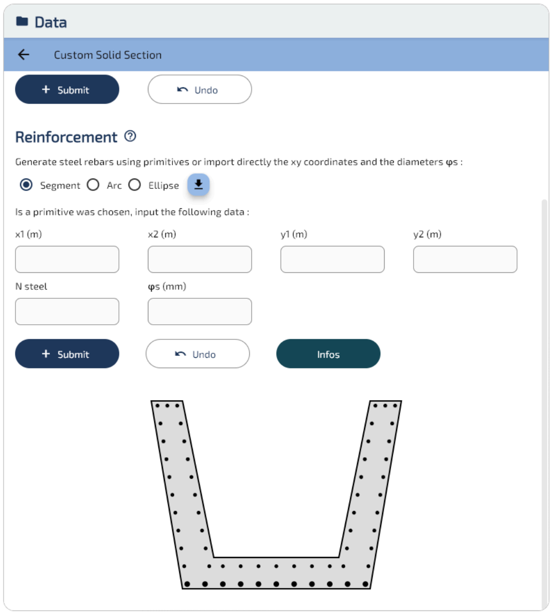


Figure 11: U-beam — geometry and reinforcement layout.

SLS — Pure bending (M_z)

Imposed loads: $N = 0$ kN, $M_z = 1500$ kN · m, $M_y = 0$

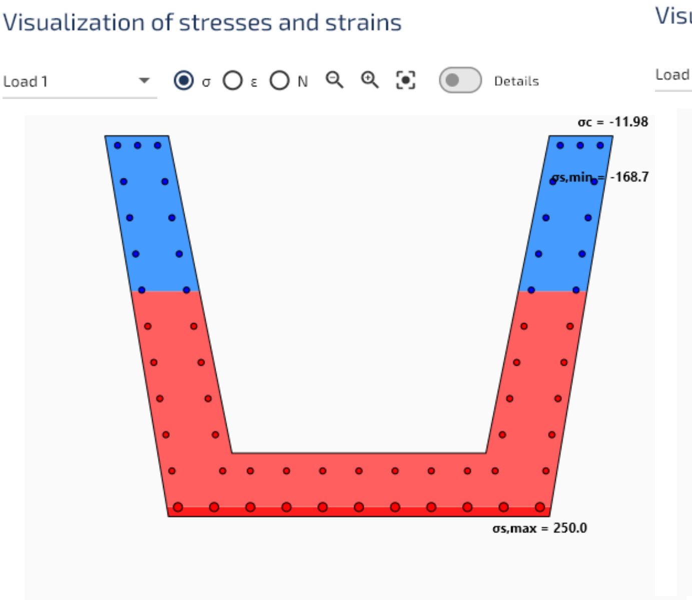


Figure 12: Stress distribution.

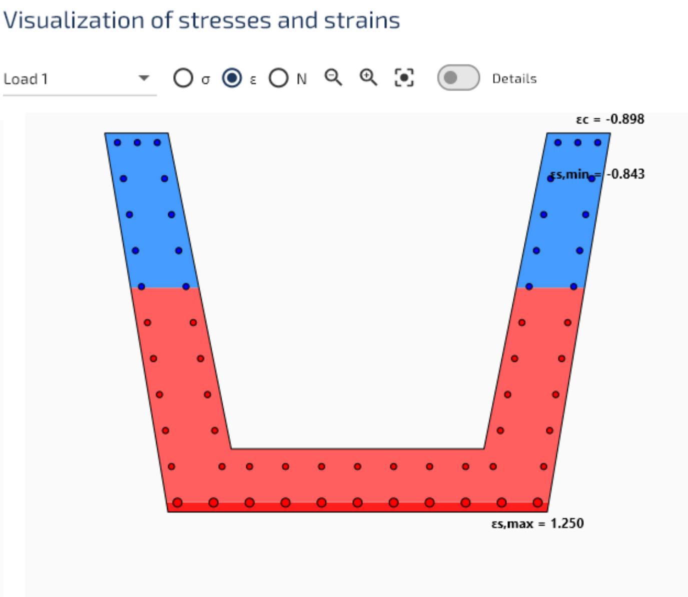


Figure 13: Strain distribution.

Stresses & strains + design

σ_c	-11.98 MPa
$\sigma_{s,\min}$	-168.65 MPa
$\sigma_{s,\max}$	250.00 MPa
ε_c	$-0.898‰$
$\varepsilon_{s,\min}$	$-0.843‰$
$\varepsilon_{s,\max}$	$1.250‰$
Pivot	A
φ_s	**17.88 mm**

Internal forces

N_c	1680.9 kN
N_t	-1680.9 kN
x_C	0.000 m
y_C	0.571 m
x_T	0.000 m
y_T	-0.321 m
z	0.892 m

Convergence

N_{iter}	4
Tol	2.21×10^{-7}
N_{int}	0.0 kN
$M_{z,\text{int}}$	1500.0 kN · m
$M_{y,\text{int}}$	0.0 kN · m
ε_0	0.434×10^{-3}
κ_x	-1.836×10^{-3}
κ_y	0.000×10^{-3}

Pivot A: the steel governs ($\sigma_{s,\max} = 250.00$ MPa $= \overline{\sigma}_s$, the BAEL allowable stress for prejudicial cracking). The required diameter is $\varphi_s = 17.88$ mm applied uniformly to all 60 bar positions.

ULS — Biaxial bending ($M_y + M_z$)

Imposed loads: $N = 0$ kN, $M_z = 2000$ kN · m, $M_y = 500$ kN · m

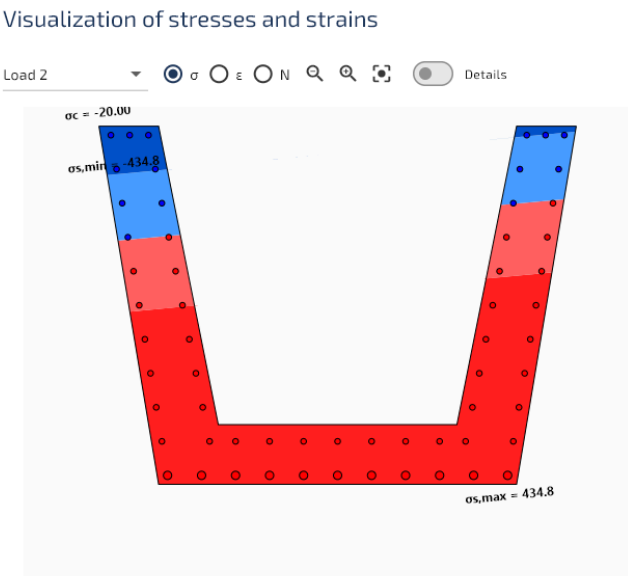


Figure 14: Stress distribution.

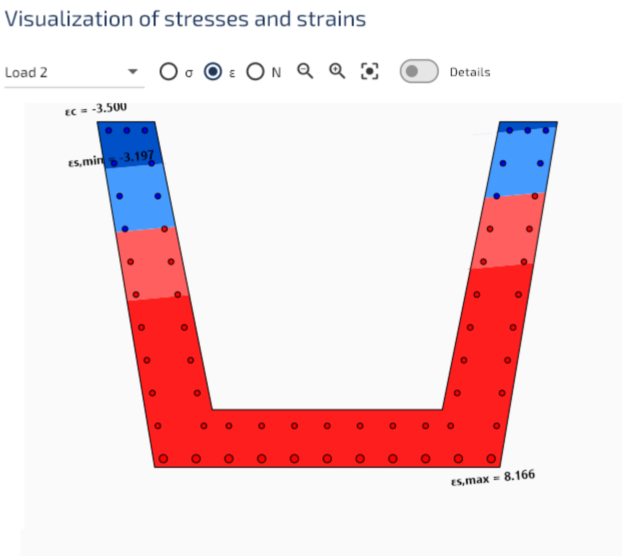


Figure 15: Strain distribution.

Stresses & strains + design

σ_c	-20.00 MPa
$\sigma_{s,\min}$	-434.78 MPa
$\sigma_{s,\max}$	434.78 MPa
ε_c	$-3.500‰$
$\varepsilon_{s,\min}$	$-3.197‰$
$\varepsilon_{s,\max}$	$8.166‰$
Pivot	B
φ_s	**13.26 mm**

Internal forces

N_c	2386.2 kN
N_t	-405.4 kN
x_C	-0.142 m
y_C	0.596 m
x_T	-0.060 m
y_T	0.191 m
z	0.413 m

Convergence

N_{iter}	39
Tol	3.29×10^{-8}
N_{int}	0.0 kN
$M_{z,\text{int}}$	2000.0 kN · m
$M_{y,\text{int}}$	500.0 kN · m
ε_0	3.693×10^{-3}
κ_x	-8.996×10^{-3}
κ_y	-0.833×10^{-3}

Pivot B: the concrete governs ($\varepsilon_c = -3.500‰ = \varepsilon_{cu}$). The required diameter is $\varphi_s = 13.26$ mm for the ULS biaxial loading.

Results validation

Internal equilibrium check

The imposed loads (N, M_y, M_z) are the **input**. SectionPro finds the bar diameter φ_s and the corresponding strain state $(\varepsilon_0, \kappa_y, \kappa_z)$ by iterative solving, then integrates stresses over the section to obtain the **internal** forces $(N_{\text{int}}, M_{y,\text{int}}, M_{z,\text{int}})$. At convergence, these must match the imposed loads:

$$N_{\text{int}} \approx N \qquad M_{y,\text{int}} \approx M_y \qquad M_{z,\text{int}} \approx M_z$$

Section	Load	N (kN)	N_{int} (kN)	M_z (kN·m)	$M_{z,\text{int}}$ (kN·m)	Δ
Hexagonal	SLS	500.0	500.0	1000.0	1000.0	0.00 %
	ULS	2000.0	2000.0	3000.0	3000.0	0.00 %
Hollow sq.	SLS	−400.0	−400.0	900.0	900.0	0.00 %
	ULS	0.0	0.0	6000.0	6000.0	0.00 %
U-beam	SLS	0.0	0.0	1500.0	1500.0	0.00 %
	ULS	0.0	0.0	2000.0	2000.0	0.00 %

Internal equilibrium is satisfied to machine precision for all six load cases — across three different geometries, three normative codes, and both linear (SLS) and nonlinear (ULS) material laws.

Cross-reference with Article #2

The table below compares the factor of safety from Article #2 (fixed reinforcement) with the required φ_s computed in this article. The reinforcement design applies a uniform φ_s to all bar positions.

Section	Load	φ_s (Art. #2)	FS (Art. #2)	Check (Art. #2)	Pivot	φ_s required
Hexagonal	SLS	25 mm	0.527	OK	A	17.6 mm
	ULS	25 mm	1.121	KO	B	25.1 mm
Hollow sq.	SLS	20 mm	0.274	OK	A	10.0 mm
	ULS	20 mm	0.634	OK	B	19.4 mm
U-beam	SLS	20/12 mm	1.209	KO	A	17.9 mm
	ULS	20/12 mm	0.436	OK	B	13.3 mm

For uniform-reinforcement sections (hexagonal and hollow square), the correlation is straightforward: FS > 1 implies $\varphi_{s,\text{required}} > \varphi_{s,\text{original}}$ and vice versa. For the U-beam, which had mixed diameters, the comparison must be made on total steel area rather than on φ_s alone.

Performance benchmark — 100,000 load cases

To demonstrate SectionPro's suitability for systematic reinforcement design, we run 100,000 load cases on **each of the three sections** defined above. The load cases combine SLS and ULS, uniaxial and biaxial bending. The benchmark measures the pure computation time, excluding UI overhead. Convergence was obtained for all 300,000 load cases.

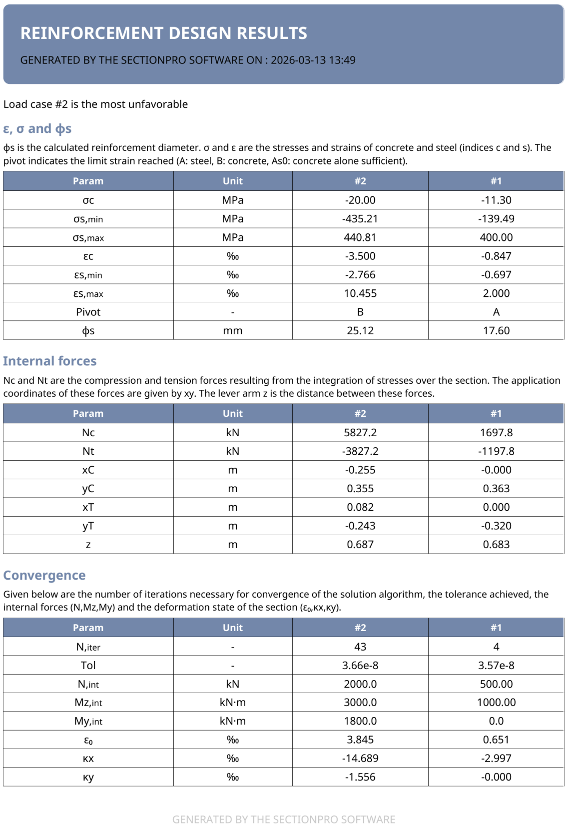
Metric	Hexagonal	Hollow square	U-beam
Load cases	100,000	100,000	100,000
Computation time	5.26 s	5.30 s	5.35 s
Rate	19,000 loads/s	18,900 loads/s	18,700 loads/s

All three sections complete in approximately 5.3 seconds for 100,000 load cases — rates of 18,700 to 19,000 designs per second. This is slower than the stress verification (Article #2), which is expected: the reinforcement design adds an outer iteration loop on φ_s, with each iteration requiring a full solve on the strain state $(\varepsilon_0, \kappa_y, \kappa_z)$.

Convergence was obtained for all 300,000 load cases, across all three geometries, normative codes, and limit states. Despite this additional layer, SectionPro designs 100,000 load cases in under 6 seconds, making it practical for systematic reinforcement design of large load envelopes.

Export

SectionPro exports results in three formats: **PDF**, **text** (fixed-width columns), and **Excel** (.xlsx). The exported data includes, per load case: stresses and strains, the failure pivot, the required bar diameter φ_s, internal forces (with centroids and lever arm), and full convergence information.



REINFORCEMENT DESIGN RESULTS

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Load case #2 is the most unfavorable

ε, σ and ϕs

ϕs is the calculated reinforcement diameter. σ and ε are the stresses and strains of concrete and steel (indices c and s). The pivot indicates the limit strain reached (A: steel, B: concrete, As0: concrete alone sufficient).

Param	Unit	#2	#1
σc	MPa	-20.00	-11.30
$\sigma s_{,min}$	MPa	-435.21	-139.49
$\sigma s_{,max}$	MPa	440.81	400.00
εc	‰	-3.500	-0.847
$\varepsilon s_{,min}$	‰	-2.766	-0.697
$\varepsilon s_{,max}$	‰	10.455	2.000
Pivot	-	B	A
ϕs	mm	25.12	17.60

Internal forces

Nc and Nt are the compression and tension forces resulting from the integration of stresses over the section. The application coordinates of these forces are given by xy. The lever arm z is the distance between these forces.

Param	Unit	#2	#1
Nc	kN	5827.2	1697.8
Nt	kN	-3827.2	-1197.8
xC	m	-0.255	-0.000
yC	m	0.355	0.363
xT	m	0.082	0.000
yT	m	-0.243	-0.320
z	m	0.687	0.683

Convergence

Given below are the number of iterations necessary for convergence of the solution algorithm, the tolerance achieved, the internal forces (N,Mz,My) and the deformation state of the section (ε_0,κx,κy).

Param	Unit	#2	#1
$N_{,iter}$	-	43	4
Tol	-	3.66e-8	3.57e-8
$N_{,int}$	kN	2000.0	500.00
$Mz_{,int}$	kN·m	3000.0	1000.00
$My_{,int}$	kN·m	1800.0	0.0
ε_0	‰	3.845	0.651
κx	‰	-14.689	-2.997
κy	‰	-1.556	-0.000

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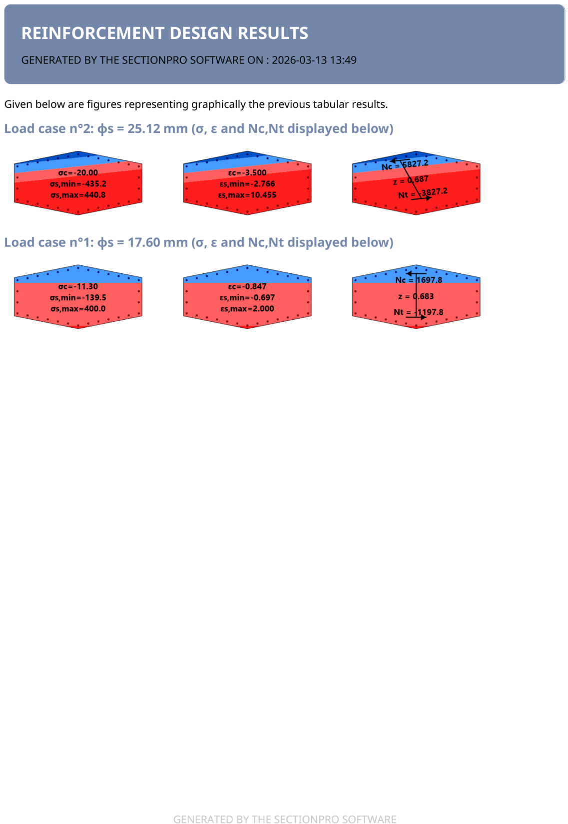
Figure 16: PDF export — page 1: results tables.



REINFORCEMENT DESIGN RESULTS

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Given below are figures representing graphically the previous tabular results.

Load case n°2: ϕs = 25.12 mm (σ, ε and Nc,Nt displayed below)

Load case n°1: ϕs = 17.60 mm (σ, ε and Nc,Nt displayed below)

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Figure 17: PDF export — page 2: figures.

Conclusion

In practice, a structural engineer typically faces two complementary problems: either verifying a section with known reinforcement — as covered in Article #2 — or determining the reinforcement required to resist a given set of loads. The reinforcement design feature addresses the second case directly. When the bar layout is known but the diameter is not yet fixed, SectionPro finds the minimum φ_s such that the section is loaded exactly to 100% of its capacity under the normative strain limits. This gives the engineer the strictly minimal reinforcement as a starting point, from which a practical bar diameter can be selected.

The results are consistent with the inverse problem formulation: internal equilibrium is satisfied to machine precision for all load cases, across three different geometries, three normative codes, and both SLS and ULS limit states. The solver converges reliably in all cases. As for performance, the benchmark of 100,000 load cases serves as an upper bound — in practice, a structural engineer typically works with a few hundred load combinations. At the measured rate of 19,000 designs per second, 500 combinations complete in under 30 milliseconds: the computation is essentially instantaneous.