

# Mechanical Properties

SectionPro Tutorial — Square section, hollow circular section & L-shaped wall

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## Introduction

The mechanical properties of a section — area, moments of inertia, centroid, torsion constant, shear areas — are the starting point of any structural analysis. This article shows how to obtain them with **SectionPro**, on three different geometries:

1. **Square section** — the simplest case, all properties can be computed analytically.
2. **Hollow circular section** — torsion and inertia remain analytical, but the shear areas require a numerical computation.
3. **L-shaped wall** — only the geometric properties are analytical. Torsion, shear and warping are purely numerical. This section illustrates the case of an asymmetric geometry ( $\alpha \neq 0$ ).

## Computed properties

SectionPro computes the following properties. The first three groups are computed for the gross section, net section (deducting voids at reinforcement locations) and transformed section (accounting for reinforcement through the modular ratio  $n$ ):

### General results

$A$  — Area  
 $(z_G, y_G)$  — Centroid  
 $P$  — Perimeter  
 $W$  — Linear weight

### Centroidal axes

$I_{zz}, I_{yy}$  — Moments of inertia  
 $v^+, v^-$  — Extreme fibers ( $y$ )  
 $w^+, w^-$  — Extreme fibers ( $z$ )

### Principal axes

$\alpha$  — Rotation angle  
 $I_1, I_2$  — Principal moments of inertia  
 $v^+, v^-, w^+, w^-$  — Extreme fibers

### Torsion & shear (FEM)

$J$  — Torsion constant  
 $A_{sy}, A_{sz}$  — Shear areas  
 $(y_T, z_T)$  — Shear center  
 $\Gamma$  — Warping constant

The torsion and shear properties require solving a differential equation using the Finite Element Method.

# Square section

## Input data

### Concrete

Side length  $a = 2.0$  m

Density  $\rho = 2.5$  t/m<sup>3</sup>

### Reinforcement

HA25 spacing 200 mm, cover 50 mm

1 layer — modular ratio  $n = 5$

## Input and results

Figure 1: Input for the square section.

Cara	Unit	Raw	Net	Homog.
A	m <sup>2</sup>	4.0000	3.9823	4.0707
z <sub>G</sub>	m	1.0000	1.0000	1.0000
y <sub>G</sub>	m	1.0000	1.0000	1.0000
P	m	8.0000	—	—
W	T/m	10.0000	—	—

Figure 2: Section properties results page.

Due to double symmetry, the centroid is at the center of the square, the principal angle is zero and both moments of inertia are equal.

## General results

	Unit	Gross	Net	Transf.
$A$	m <sup>2</sup>	4.0000	3.9823	4.0707
$z_G$	m	1.0000	1.0000	1.0000
$y_G$	m	1.0000	1.0000	1.0000
$P$	m	8.0000	—	—
$W$	T/m	10.0000	—	—

## Bending

### 3.2.2.1 Centroidal axes

	Unit	Gross	Net	Transf.
$I_{zz}$	m <sup>4</sup>	1.3333	1.3226	1.3761
$I_{yy}$	m <sup>4</sup>	1.3333	1.3226	1.3761
$v^+$	m	1.0000	1.0000	1.0000
$v^-$	m	1.0000	1.0000	1.0000
$w^+$	m	1.0000	1.0000	1.0000
$w^-$	m	1.0000	1.0000	1.0000

### 3.2.2.2 Principal axes

	Unit	Gross	Net	Transf.
$I_1$	m <sup>4</sup>	1.3333	1.3226	1.3761
$I_2$	m <sup>4</sup>	1.3333	1.3226	1.3761
$v^+$	m	1.0000	1.0000	1.0000
$v^-$	m	1.0000	1.0000	1.0000
$w^+$	m	1.0000	1.0000	1.0000
$w^-$	m	1.0000	1.0000	1.0000
$\alpha$	°	0.00	0.00	0.00

## Torsion and shear (FEM)

Due to double symmetry, the shear center coincides with the centroid ( $y_T = z_T = y_G = z_G = 1.0$  m). The warping is negligible ( $\Gamma \approx 0$ ). The ratio  $A_{sz}/A = 3.33/4.00 \approx 0.83$ , typical of a solid section.

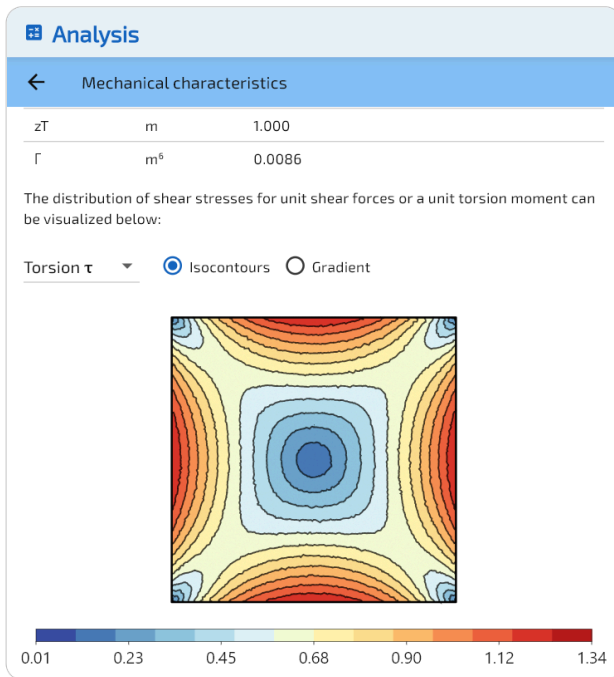


Figure 3: Torsion stresses  $\tau$  — maximum at mid-side.

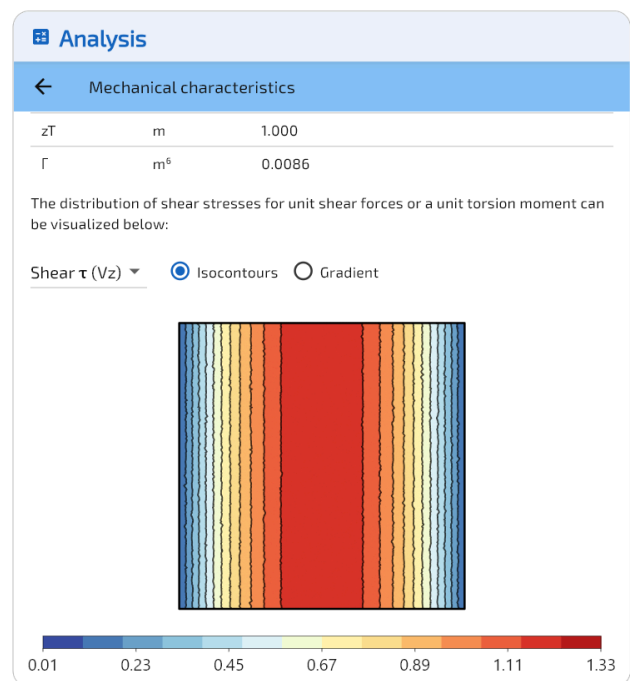


Figure 4: Shear stresses.

	$J$	$A_{sz}$	$A_{sy}$	$y_T$	$z_T$	$\Gamma$
Unit	m <sup>4</sup>	m <sup>2</sup>	m <sup>2</sup>	m	m	m <sup>6</sup>
Value	2.2492	3.3333	3.3333	1.0000	1.0000	0.0086

# Hollow circular section

## Input data

### Concrete

Outer diameter  $D = 2.0$  m

Wall thickness  $e = 0.3$  m

Density  $\rho = 2.5$  t/m<sup>3</sup>

### Reinforcement

24 HA20, cover 50 mm

1 layer — modular ratio  $n = 5$

## Input and results

Data

← Hollow Circular Section

**Concrete**

Outer diameter (m)  Wall thickness (m)

**Reinforcement** ⓘ

Mode: count

Number of rebars  Bar diameter (mm)  Concrete cover (mm)  Layers (1 or 2)

Submit
Infos

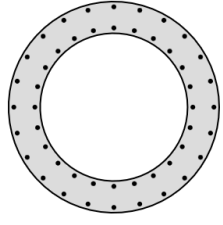


Figure 5: Input for the hollow circular section.

Analysis

← Mechanical characteristics

**General results**

The area  $A$ , perimeter  $P$ , weight per linear meter  $W$  and the coordinates of the center of gravity ( $z_G$ ,  $y_G$ ) are given below:

Cara	Unit	Raw	Net	Homog.
$A$	m <sup>2</sup>	1.6022	1.5871	1.6625
$z_G$	m	1.000	1.000	1.000
$y_G$	m	1.000	1.000	1.000
$P$	m	6.283	-	-
$W$	T/m	4.005	-	-

*$z_G$  and  $y_G$  are given respectively with respect to the extreme fiber on the left and at the bottom of the section.*

**Centroidal reference frame**

The central reference frame is positioned at the center of gravity of the gross section. The  $z$ -axis is horizontal, oriented to the right and the  $y$ -axis is vertical, oriented upwards.

The moments of inertia  $I_{zz}$  and  $I_{yy}$ , as well as the distances of the extreme fibers at the top ( $v^+$ ), at the bottom ( $v^-$ ), to the right ( $w^+$ ) and to the left ( $w^-$ ) with respect to this reference frame are given below:

Cara	Unit	Raw	Net	Homog.
$I_{zz}$	m <sup>4</sup>	0.5968	0.5913	0.6189
$I_{yy}$	m <sup>4</sup>	0.5968	0.5913	0.6189
$v^+$	m	1.000	1.000	1.000
$v^-$	m	1.000	1.000	1.000
$w^+$	m	1.000	1.000	1.000
$w^-$	m	1.000	1.000	1.000

Figure 6: Section properties results page.

Due to circular symmetry, the moments of inertia are equal and the principal angle is indeterminate (displayed as  $0^\circ$ ).

## General results

	Unit	Gross	Net	Transf.
$A$	m <sup>2</sup>	1.6022	1.5871	1.6625
$z_G$	m	1.0000	1.0000	1.0000
$y_G$	m	1.0000	1.0000	1.0000
$P$	m	6.2832	—	—
$W$	T/m	4.0055	—	—

## Bending

### 4.2.2.1 Centroidal axes

	Unit	Gross	Net	Transf.
$I_{zz}$	m <sup>4</sup>	0.5968	0.5913	0.6189
$I_{yy}$	m <sup>4</sup>	0.5968	0.5913	0.6189
$v^+$	m	1.0000	1.0000	1.0000
$v^-$	m	1.0000	1.0000	1.0000
$w^+$	m	1.0000	1.0000	1.0000
$w^-$	m	1.0000	1.0000	1.0000

### 4.2.2.2 Principal axes

	Unit	Gross	Net	Transf.
$I_1$	m <sup>4</sup>	0.5968	0.5913	0.6189
$I_2$	m <sup>4</sup>	0.5968	0.5913	0.6189
$v^+$	m	1.0000	1.0000	1.0000
$v^-$	m	1.0000	1.0000	1.0000
$w^+$	m	1.0000	1.0000	1.0000
$w^-$	m	1.0000	1.0000	1.0000
$\alpha$	°	0.00	0.00	0.00

## Torsion and shear (FEM)

Due to axial symmetry, the shear center coincides with the centroid ( $y_T = z_T = y_G = z_G = 1.0$  m) and the warping is zero ( $\Gamma = 0$ ). The ratio  $A_{sz}/A = 0.84/1.60 \approx 0.53$ : the hollow section is less efficient in shear than a solid section.

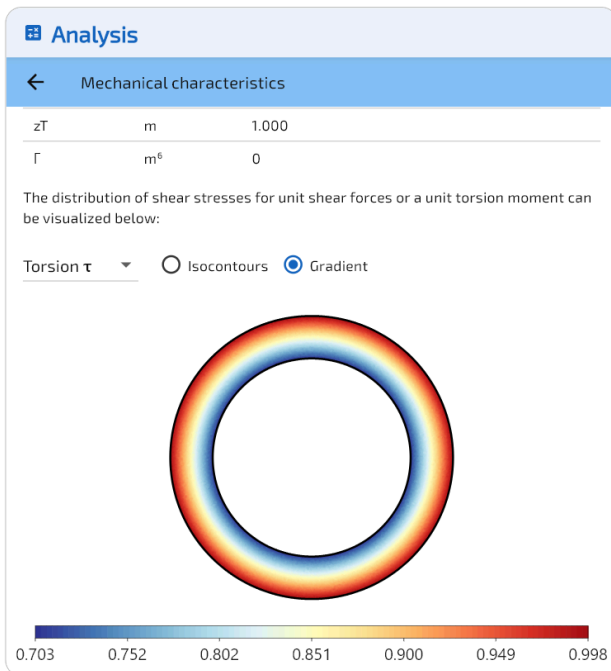


Figure 7: Torsion stresses  $\tau$  — maximum on the outer contour.

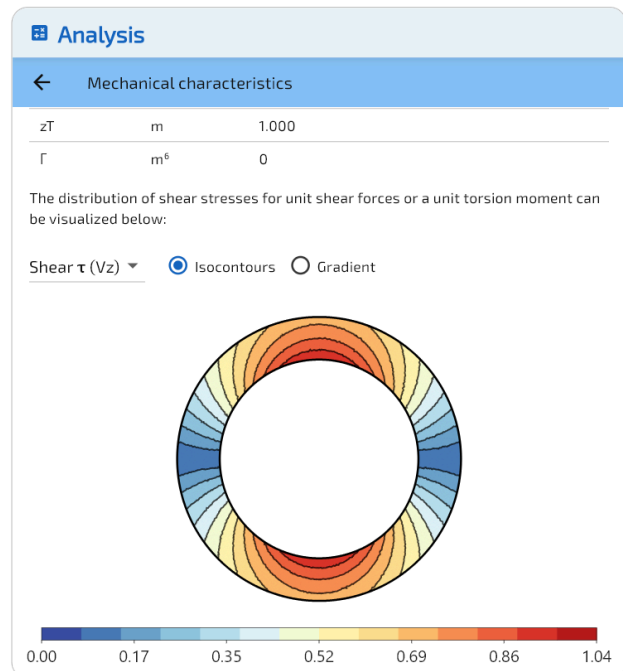


Figure 8: Shear stresses.

	$J$	$A_{sz}$	$A_{sy}$	$y_T$	$z_T$	$\Gamma$
Unit	m <sup>4</sup>	m <sup>2</sup>	m <sup>2</sup>	m	m	m <sup>6</sup>
Value	1.1936	0.8422	0.8422	1.0000	1.0000	0.0000

# L-shaped wall

## Input data

### Concrete

L-shape — width 2.0 m, height 2.0 m  
 Thickness  $t = 0.3$  m  
 Density  $\rho = 2.5$  t/m<sup>3</sup>

### Reinforcement

HA20 spacing 200 mm, cover 40 mm  
 1 layer — modular ratio  $n = 5$

## Input and results

Figure 9: Input for the L-shaped wall.

**General results**

The area A, perimeter P, weight per linear meter W and the coordinates of the center of gravity (z<sub>G</sub>, y<sub>G</sub>) are given below:

Cara	Unit	Raw	Net	Homog.
A	m <sup>2</sup>	1.1100	1.0974	1.1603
z <sub>G</sub>	m	0.609	0.609	0.610
y <sub>G</sub>	m	0.609	0.609	0.610
P	m	8.000	-	-
W	T/m	2.775	-	-

*z<sub>G</sub> and y<sub>G</sub> are given respectively with respect to the extreme fiber on the left and at the bottom of the section.*

**Centroidal reference frame**

The central reference frame is positioned at the center of gravity of the gross section. The z-axis is horizontal, oriented to the right and the y-axis is vertical, oriented upwards.

Figure 10: Section properties results page.

Since both flanges have the same length,  $I_{zz} = I_{yy}$  and the principal angle is exactly  $\alpha = 45^\circ$ .

## General results

	Unit	Gross	Net	Transf.
A	m <sup>2</sup>	1.1100	1.0974	1.1603
z <sub>G</sub>	m	0.6095	0.6093	0.6100
y <sub>G</sub>	m	0.6095	0.6093	0.6100
P	m	8.0000	—	—
W	T/m	2.7750	—	—

## Bending

### 5.2.2.1 Centroidal axes

	Unit	Gross	Net	Transf.
$I_{zz}$	m <sup>4</sup>	0.4030	0.3981	0.4225
$I_{yy}$	m <sup>4</sup>	0.4030	0.3981	0.4225
$v^+$	m	1.3905	1.3907	1.3900
$v^-$	m	0.6095	0.6093	0.6100
$w^+$	m	1.3905	1.3907	1.3900
$w^-$	m	0.6095	0.6093	0.6100

### 5.2.2.2 Principal axes

	Unit	Gross	Net	Transf.
$I_1$	m <sup>4</sup>	0.6373	0.6297	0.6679
$I_2$	m <sup>4</sup>	0.1687	0.1666	0.1771
$v^+$	m	1.4142	1.4142	1.4142
$v^-$	m	1.4142	1.4142	1.4142
$w^+$	m	0.7644	0.7644	0.7644
$w^-$	m	0.8619	0.8619	0.8619
$\alpha$	°	45.00	45.00	45.00

## Torsion and shear (FEM)

The shear center ( $y_T = z_T = 0.16$  m) is offset toward the re-entrant corner, far from the centroid ( $y_G = z_G = 0.61$  m). The warping is significant ( $\Gamma = 0.009$  m<sup>6</sup>). The torsion constant  $J = 0.032$  m<sup>4</sup> is very low — typical of an open thin-walled section. The ratio  $A_{sz}/A = 0.50/1.11 \approx 0.45$ .

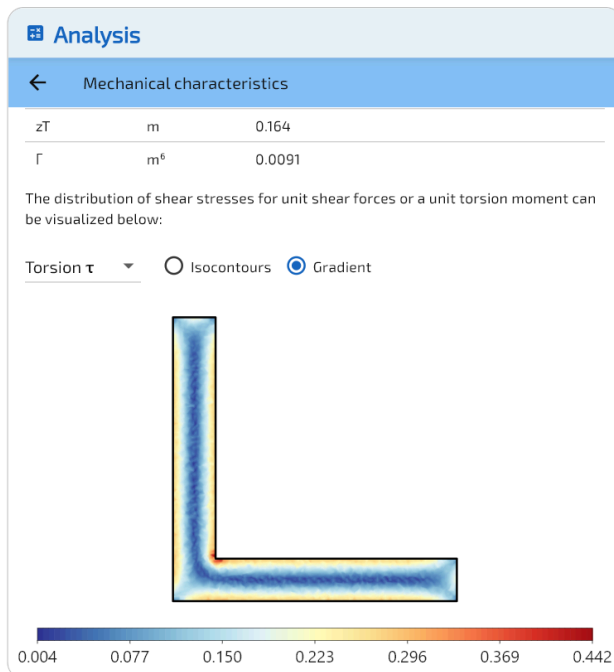


Figure 11: Torsion stresses  $\tau$  — stress concentration at the re-entrant corner. Shear center offset.

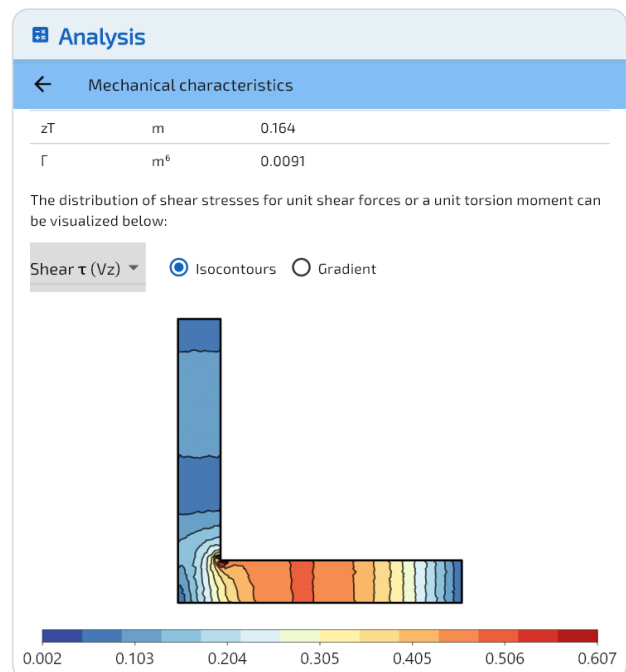


Figure 12: Shear stresses.

	$J$	$A_{sz}$	$A_{sy}$	$y_T$	$z_T$	$\Gamma$
Unit	m <sup>4</sup>	m <sup>2</sup>	m <sup>2</sup>	m	m	m <sup>6</sup>
Value	0.0322	0.5037	0.5037	0.1637	0.1637	0.0091

# Results validation

SectionPro results are validated in two ways: by comparison with **analytical formulas** (when available) and by **cross-validation** with a reference software using an independent finite element solver.

## Analytical formulas

**Square section** ( $a = 2.0$  m)

$$A = a^2 = 4.0000 \quad I = \frac{a^4}{12} = 1.3333 \quad J = 0.1406a^4 = 2.2489 \quad A_s = \frac{5}{6}A = 3.3333$$

The torsion constant is obtained from Saint-Venant series:

$$J = \frac{a^4}{3} \left[ 1 - \frac{192}{\pi^5} \sum_{n=1,3,5,\dots}^{\infty} \frac{\tanh(n\pi/2)}{n^5} \right]$$

**Hollow circular section** ( $R = 1.0$  m,  $r = 0.7$  m)

$$A = \pi(R^2 - r^2) = 1.6022 \quad I = \frac{\pi}{4}(R^4 - r^4) = 0.5968 \quad J = \frac{\pi}{2}(R^4 - r^4) = 1.1936$$

The shear areas do not have a simple closed-form expression; the differential equation must be solved numerically.

**L-shaped wall** ( $L = 2.0$  m,  $t = 0.3$  m)

By decomposition (flange  $2.0 \times 0.3$  + web  $0.3 \times 1.7$ ) and the Parallel Axis Theorem:

$$A = 1.1100 \quad z_G = y_G = 0.6095 \quad I_{zz} = I_{yy} = 0.4030 \quad \alpha = 45^\circ$$

There is no exact analytical formula for torsion, shear and warping. Vlasov beam theory (open thin-walled sections) provides an order of magnitude:  $J \approx \frac{1}{3}(2L - t)t^3 = 0.0333$  m<sup>4</sup> and the shear center is located approximately at the intersection of the flange mid-lines ( $y_T \approx z_T \approx t/2 = 0.15$  m). These estimates assume a thickness that is infinitely small compared to the flange length; here  $t/L = 15\%$ , and thickness effects — particularly the stiffening at the junction corner — shift the actual values away from this simplified model.

## Net and transformed sections

For a section reinforced with  $N$  steel bars of area  $A_{s,i}$  at coordinates  $(z_{s,i}, y_{s,i})$ , with a modular ratio  $n$ :

$$A_{\text{net}} = A - \sum_{i=1}^N A_{s,i} \quad A_{\text{hom}} = A + (n-1) \sum_{i=1}^N A_{s,i}$$

The centroid shifts slightly (analogous formula for  $z_G$ ):

$$y_{G,\text{net}} = \frac{A \cdot y_G - \sum_i A_{s,i} y_{s,i}}{A_{\text{net}}} \quad y_{G,\text{hom}} = \frac{A \cdot y_G + (n-1) \sum_i A_{s,i} y_{s,i}}{A_{\text{hom}}}$$

The moment of inertia is obtained using the Parallel Axis Theorem, accounting for the offset  $\Delta y_G$  between the centroid of the considered section and that of the gross section:

$$I_{zz,\text{net}} = I_{zz} + A(\Delta y_G)^2 - \sum_i A_{s,i} (y_{s,i} - y_{G,\text{net}})^2$$

$$I_{zz,\text{hom}} = I_{zz} + A(\Delta y_G)^2 + (n-1) \sum_i A_{s,i} (y_{s,i} - y_{G,\text{hom}})^2$$

## Validation — Bending properties

The analytical formulas above were applied to all three sections using the exact reinforcement coordinates exported by SectionPro. All results match.

Section	Property	Gross	$\Delta$	Net	$\Delta$	Transf.	$\Delta$
Square	$A$ (m <sup>2</sup> )	4.0000	0.00 %	3.9823	0.00 %	4.0707	0.00 %
	$z_G, y_G$ (m)	1.0000	0.00 %	1.0000	0.00 %	1.0000	0.00 %
	$I_{zz}, I_{yy}$ (m <sup>4</sup> )	1.3333	0.00 %	1.3226	0.00 %	1.3761	0.00 %
Hollow circ.	$A$ (m <sup>2</sup> )	1.6022	0.00 %	1.5871	0.00 %	1.6625	0.00 %
	$z_G, y_G$ (m)	1.0000	0.00 %	1.0000	0.00 %	1.0000	0.00 %
	$I_{zz}, I_{yy}$ (m <sup>4</sup> )	0.5968	0.00 %	0.5913	0.00 %	0.6189	0.00 %
L-shaped wall	$A$ (m <sup>2</sup> )	1.1100	0.00 %	1.0974	0.00 %	1.1603	0.00 %
	$z_G, y_G$ (m)	0.6095	0.00 %	0.6093	0.00 %	0.6100	0.00 %
	$I_{zz}, I_{yy}$ (m <sup>4</sup> )	0.4030	0.00 %	0.3981	0.00 %	0.4225	0.00 %

## Validation — Torsion and shear (cross-validation)

The torsion and shear properties, computed by the Finite Element Method, are compared to a reference software using an independent solver.

Section	Property	Analytical	SectionPro	$\Delta$	Ref.	$\Delta$
Square	$J$ (m <sup>4</sup> )	2.2489	2.2492	0.01 %	2.2585	0.41 %
	$A_{sz}, A_{sy}$ (m <sup>2</sup> )	3.3333	3.3333	0.00 %	3.3355	0.07 %
	$y_T, z_T$ (m)	1.0000	1.0000	0.00 %	1.0000	0.00 %
Hollow circ.	$J$ (m <sup>4</sup> )	1.1936	1.1936	0.00 %	1.1920	0.13 %
	$A_{sz}, A_{sy}$ (m <sup>2</sup> )	—	0.8422	—	0.8418	—
	$y_T, z_T$ (m)	1.0000	1.0000	0.00 %	1.0000	0.00 %
L-shaped wall	$J$ (m <sup>4</sup> )	—	0.0322	—	0.0328	—
	$A_{sz}$ (m <sup>2</sup> )	—	0.5037	—	0.5054	—
	$A_{sy}$ (m <sup>2</sup> )	—	0.5037	—	0.5024	—
	$y_T, z_T$ (m)	—	0.1637	—	0.1639	—

**L-shaped wall** — Vlasov beam theory ( $J \approx 0.033$  m<sup>4</sup>,  $y_T \approx 0.15$  m) provides a comparable order of magnitude, but remains an approximation since it considers segments with zero thickness (whereas  $t/L = 15\%$ ).

## Conclusion

Section	Validation	Bending error	Torsion error (ref.)
Square	Analytical	0.00 %	0.41 %
Hollow circ.	Analytical + reference ( $A_{sy}$ , $A_{sz}$ )	0.00 %	0.13 %
L-shaped wall	Analytical + reference ( $J$ , $A_{sy}$ , $A_{sz}$ , $y_T$ , $z_T$ )	0.00 %	1.86 %

The bending properties (area, centroid, moments of inertia) are reproduced with perfect accuracy on all three geometries, for the gross, net and transformed sections (0.00 % deviation from analytical formulas).

The torsion and shear properties, computed by the Finite Element Method, depend on mesh refinement. The cross-validation with a reference software shows very good agreement between the two solvers. SectionPro nevertheless exhibits better convergence, as evidenced by its exact match with the analytical torsion and shear solutions when they exist.