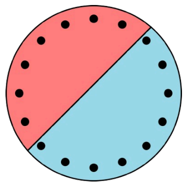

Interaction Surface

SectionPro Tutorial: 3D resistance domains under biaxial bending

BridgeKernel · 2026

Introduction

Verifying a reinforced concrete section under combined axial force and bending is straightforward when the bending is uniaxial: a single interaction diagram (N, M) captures the full resistance domain. Under biaxial bending, however, the resistance depends simultaneously on N, M_z, and M_y, and no single 2D curve is sufficient. The complete resistance domain becomes a three-dimensional surface in the $\left(N, M_z, M_y\right)$ space.

SectionPro computes this interaction surface for each limit state defined by the user. A load point inside the surface means the section is safe; a point outside means the capacity is exceeded. The surface is displayed in an interactive 3D viewer with rotation, zoom, and pan controls, and can be exported to PDF, Excel, and text formats.

This article demonstrates the interaction surface analysis on two geometries and two normative codes: an **octagonal section** (Eurocode 2) and an **elliptical section** (ACI 318), each analyzed at both ULS and SLS.

Computed results

SectionPro reports three categories of results for each interaction surface:

3D resistance domain

Interactive triangulated surface
One surface per limit state
Adjustable mesh resolution
Rotation, zoom, pan controls
Adjustable surface opacity

Bounding box

$N_{\min}, N_{\max}$: axial force range
$M_{z,\min}, M_{z,\max}$: moment range
$M_{y,\min}, M_{y,\max}$: moment range
Extreme capacity per direction

Exports

PDF: bounding box + 3D views
XLS: mesh coordinates + triangles
TXT: mesh coordinates (columns)

Test scenarios

Octagonal section (Eurocode 2). At ULS, the concrete follows a parabola-rectangle law with $\gamma_c = 1.50$ and the steel is elastoplastic with strain hardening ($\gamma_s = 1.15$). At SLS, both materials are linear elastic with user-defined allowable stresses.

Elliptical section (ACI 318). At ULS, the concrete follows the Whitney equivalent rectangular stress block. The section resistance is reduced by φ-factors that depend on the strain state ($\varphi = 0.65$ to 0.90), with an additional $\varphi_N = 0.80$ cap on the maximum compressive force. For reinforcement ratios below 1%, an architectural reduction further scales the block stress. At SLS, both materials are linear elastic with user-defined allowable stresses.

Octagonal section (Eurocode 2)

Input data

Concrete

- Octagonal cross section
- $b_1 = 2.00$ m, $b_2 = 0.50$ m
- $h_1 = 1.00$ m, $h_2 = 0.60$ m

Reinforcement

- 48 bars, uniform spacing 150 mm
- Diameter $\varphi = 32$ mm
- Cover 50 mm, 1 layer

Material laws (EC2)

- Concrete C30/37: $f_{ck} = 30$ MPa
- Steel B500B: $f_{yk} = 500$ MPa

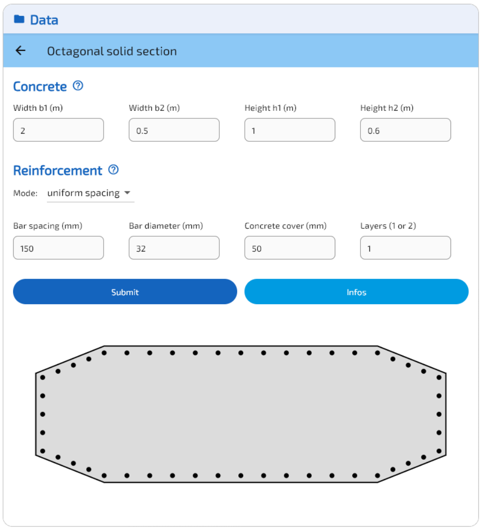

Figure 1: Octagonal section: geometry and reinforcement layout.

ULS (Fundamental)

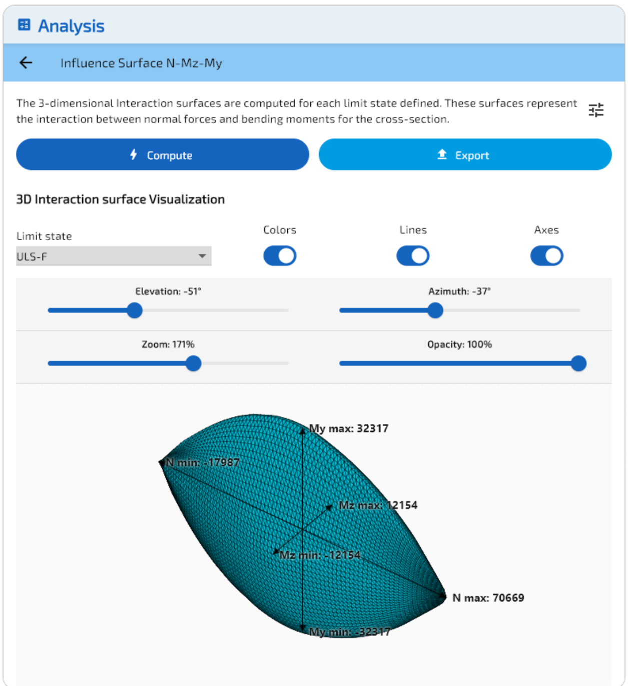

Figure 2: ULS interaction surface, colored triangles.

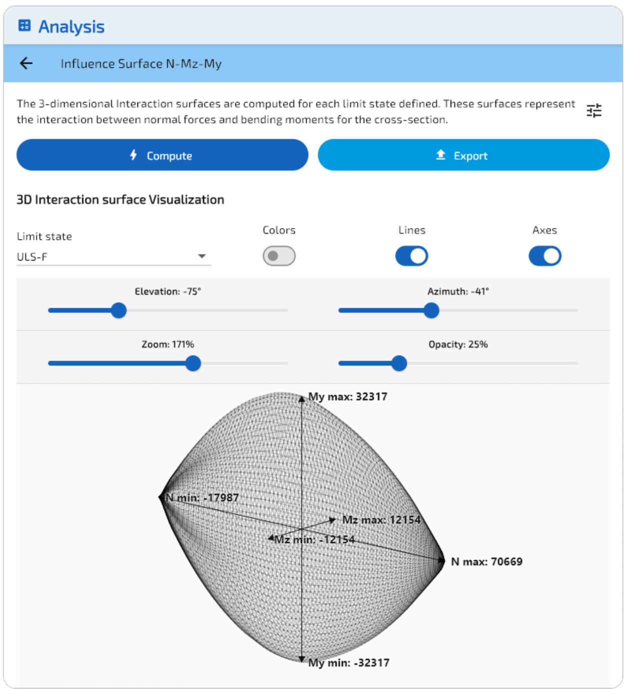

Figure 3: ULS interaction surface, wireframe view.

SLS (Characteristic)

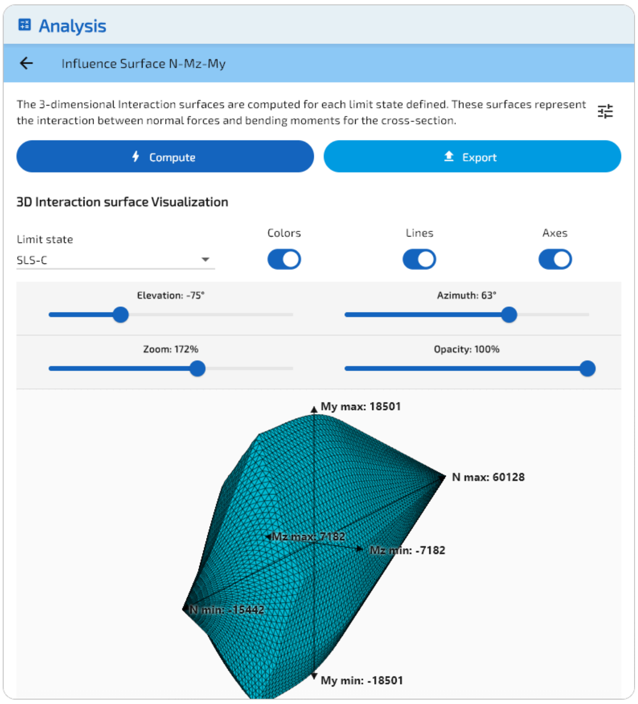

Figure 4: SLS interaction surface, colored triangles.

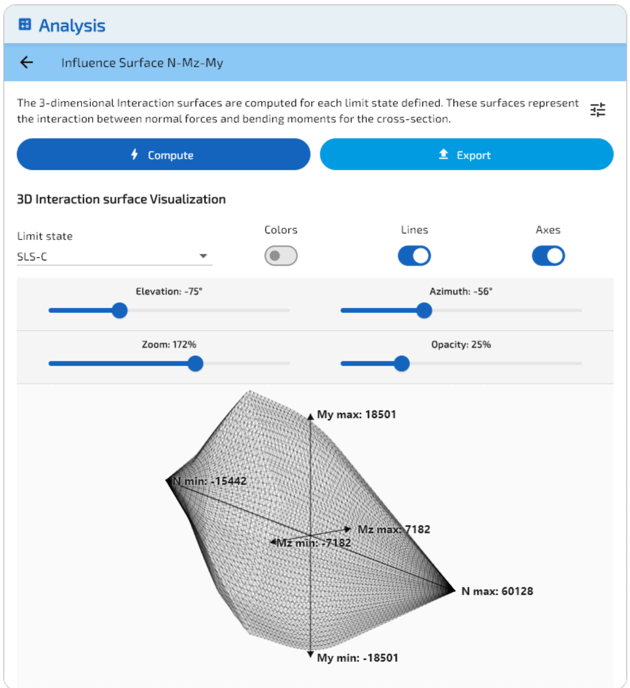

Figure 5: SLS interaction surface, wireframe view.

Bounding box

	Axial force N (kN)	Moment M_z (kN·m)	Moment M_y (kN·m)
ULS min	−17987	−12154	−32317
ULS max	70669	12154	32317
SLS min	−15442	−7182	−18501
SLS max	60128	7182	18501

- The section is wider than it is tall ($b_1 = 2.00$ m, $h_1 = 1.00$ m), so M_y extents are significantly larger than M_z: the strong axis is the vertical one (transverse bending).
- The ULS surface is larger than the SLS surface in all directions, since the nonlinear EC2 parabola-rectangle law with $\gamma_c = 1.50$ provides greater capacity than the linear elastic SLS law.

Elliptical section (ACI 318)

Input data

Concrete

- Elliptical cross section
- Width = 3.00 m (strong axis)
- Height = 2.00 m (weak axis)

Reinforcement

- 40 bars along the perimeter
- Diameter $\varphi = 40$ mm
- Cover 50 mm, 1 layer

Material laws (ACI 318)

- Concrete: $f'_c = 30$ MPa
- Steel: $f_y = 500$ MPa

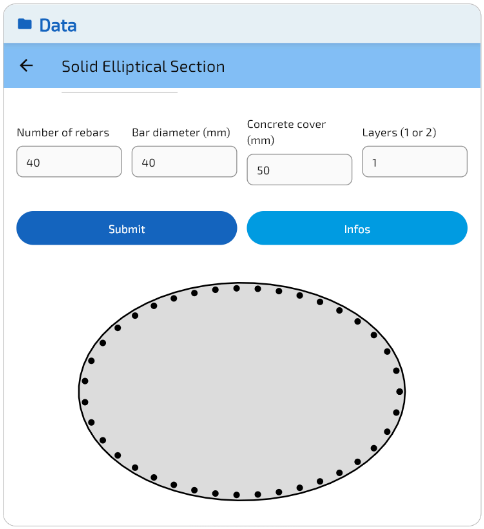

Figure 6: Elliptical section: geometry and reinforcement layout.

At ULS, the concrete stress distribution is replaced by the Whitney equivalent rectangular block, where $\beta_1 = 0.832$ relates the block depth to the neutral axis depth.

The raw interaction surface is then weighted by φ-factors that depend on the strain state at each point: $\varphi = 0.90$ for tension-controlled sections and $\varphi = 0.65$ for compression-controlled sections, with a cap $\varphi_N = 0.80$ on the maximum axial compression. Since the reinforcement ratio ($\rho = 1.07\%$) exceeds the 1% architectural threshold, no further reduction is applied to the concrete block stress.

ULS

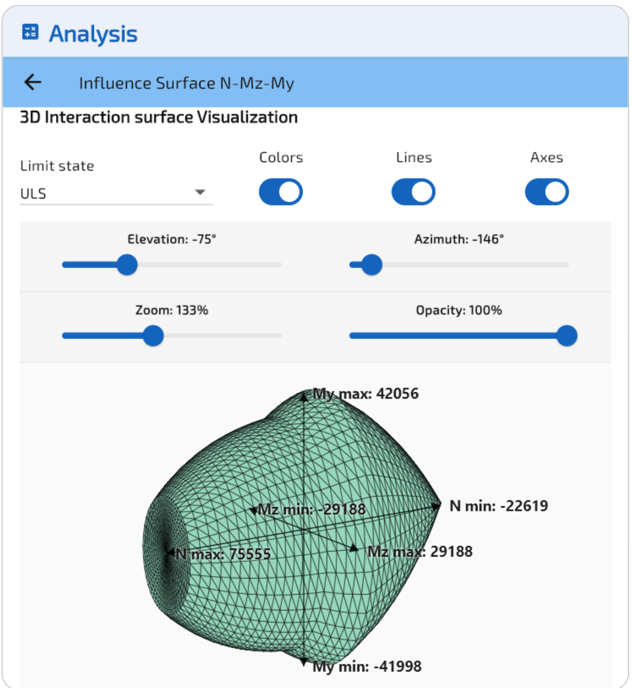

Figure 7: ULS interaction surface, colored triangles.

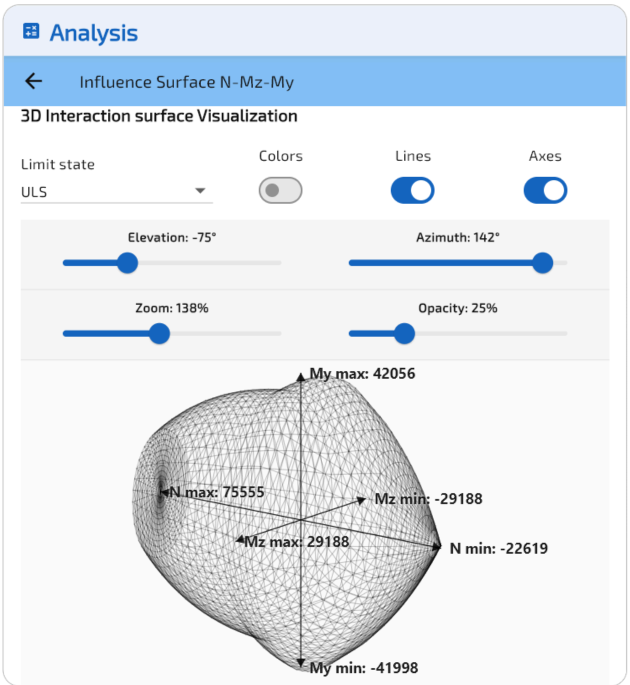

Figure 8: ULS interaction surface, wireframe view.

SLS

At SLS, the Whitney block and φ-factors no longer apply. Both concrete and steel follow linear elastic laws, limited by user-defined allowable stresses ($\sigma_c = 11.5$ MPa, $\sigma_s = 250$ MPa). The low allowable concrete stress compared to the ULS block stress ($0.85 f'_c = 25.5$ MPa) explains the smaller SLS surface.

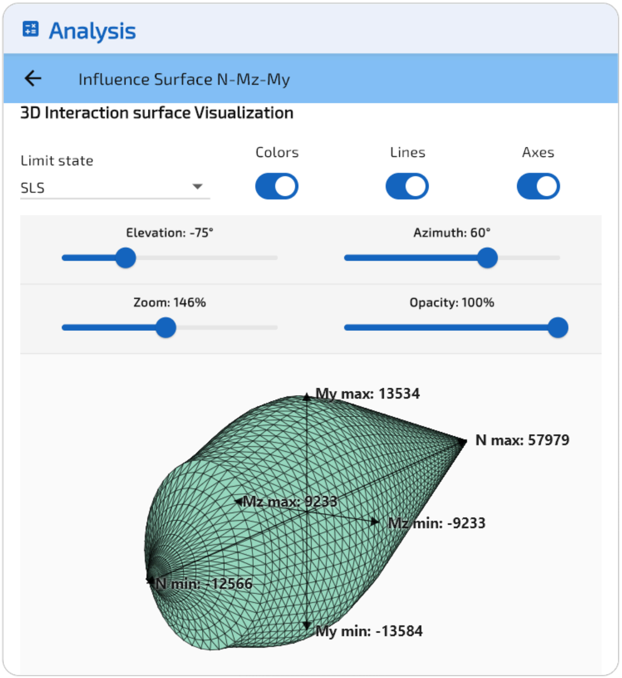

Figure 9: SLS interaction surface, colored triangles.

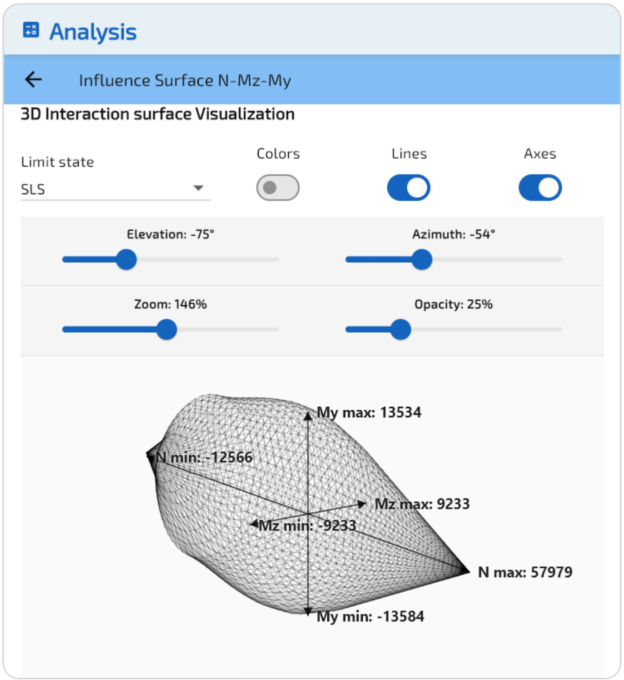

Figure 10: SLS interaction surface, wireframe view.

Bounding box

	Axial force N (kN)	Moment M_z (kN·m)	Moment M_y (kN·m)
ULS min	−22619	−29188	−41998
ULS max	75555	29188	42056
SLS min	−12566	−9233	−13584
SLS max	57979	9233	13534

- The section is wider than it is tall (3.00×2.00 m), so M_y extents are larger than M_z, but the difference is less pronounced than for the octagonal section and the surface retains a fairly round shape due to the smooth elliptical geometry.
- The ULS surface is larger than the SLS surface in compression, as expected. The ACI φ-factors ($\varphi = 0.65$ for compression-controlled, $\varphi_N = 0.80$) reduce the nominal capacity by an effective factor of 0.52, and with $\rho = 1.07\%$ the full Whitney block stress ($0.85 f'_c = 25.5$ MPa) is mobilized. At SLS, the concrete is limited to the allowable stress ($\sigma_c = 11.5$ MPa), resulting in a smaller compressive capacity.

Export

SectionPro exports the interaction surface data in three formats. The export dialog allows selecting which limit states to include. The PDF report contains the bounding box table and two 3D views of the surface from different angles. The Excel file provides two sheets per limit state: one with the mesh node coordinates (N, M_z, M_y) and one with the triangle connectivity, so the surface can be reconstructed in any external tool. A text export is also available, listing the node coordinates in fixed-width columns.

INFLUENCE SURFACE CALCULATION RESULTS

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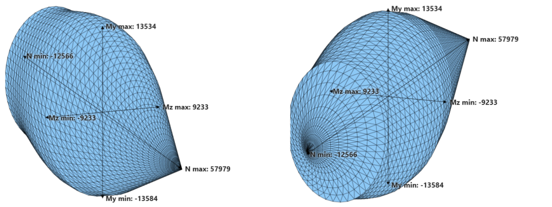
Below are figures allowing 3D visualization of the resistance domains of the studied reinforced concrete section.
The bounding boxes represent the extreme values of the resisting forces.

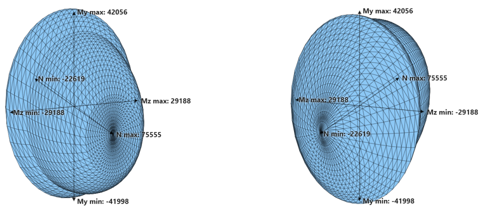
SLS : Serviceability limit state

Bounding box	Axial force N (kN)	Moment Mz (kN·m)	Moment My (kN·m)
Minimum value	-12566	-9232.7	-13584
Maximum value	57979	9232.7	13534



ULS : Ultimate limit state

Bounding box	Axial force N (kN)	Moment Mz (kN·m)	Moment My (kN·m)
Minimum value	-22619	-29188	-41998
Maximum value	75555	29188	42056



GENERATED BY THE SECTIONPRO SOFTWARE

Figure 11: PDF export: bounding box and 3D views.

INFLUENCE SURFACE CALCULATION RESULTS
GENERATED BY THE SECTIONPRO SOFTWARE ON : 2026-03-16 22:32

INFLUENCE SURFACE NODES

ID	SLS N	SLS Mz	SLS My	ULS N	ULS Mz	ULS My
	kN	kN·m	kN·m	kN	kN·m	kN·m
1	-12566.37	0.00	455.54	-22619.47	0.00	819.97
2	-11126.67	735.28	403.35	-19892.39	2557.77	811.22
3	-11126.67	729.70	297.26	-19892.39	2516.78	108.24
4	-11126.67	714.06	192.56	-19892.39	2391.21	-557.84
5	-11126.67	689.11	90.35	-19892.39	2204.62	-1146.42
6	-11126.67	655.70	-8.52	-19892.39	1989.45	-1625.51
7	-11126.67	613.41	-104.44	-19892.39	1741.43	-2023.96
8	-11126.67	562.89	-196.03	-19892.39	1489.98	-2328.95
9	-11126.67	501.17	-280.80	-19892.39	1275.81	-2547.72
10	-11126.67	429.11	-355.88	-19892.39	926.37	-2780.14
11	-11126.67	348.12	-422.33	-19892.39	704.46	-2900.77
12	-11126.67	257.87	-478.01	-19892.39	455.63	-2996.54
13	-11126.67	158.72	-517.58	-19892.39	281.52	-3046.76
14	-11126.67	53.15	-534.98	-19892.39	78.28	-3072.02
15	-11126.67	-54.09	-534.86	-19892.39	-78.28	-3072.02
16	-11126.67	-159.62	-517.22	-19892.39	-281.52	-3046.76
17	-11126.67	-258.70	-477.68	-19892.39	-455.63	-2996.54
18	-11126.67	-348.90	-421.81	-19892.39	-704.46	-2900.77
19	-11126.67	-429.75	-355.24	-19892.39	-926.37	-2780.14
20	-11126.67	-501.72	-280.06	-19892.39	-1275.81	-2547.72
21	-11126.67	-563.41	-195.32	-19892.39	-1489.98	-2328.95
22	-11126.67	-613.84	-103.63	-19892.39	-1741.43	-2023.96
23	-11126.67	-656.03	-7.68	-19892.39	-1989.45	-1625.51
24	-11126.67	-689.34	91.23	-19892.39	-2204.62	-1146.42
25	-11126.67	-714.20	193.48	-19892.39	-2391.21	-557.84
26	-11126.67	-729.84	298.18	-19892.39	-2516.78	108.24
27	-11126.67	-735.33	404.28	-19892.39	-2557.77	811.22
28	-11126.67	-732.29	510.75	-19892.39	-2511.26	1542.12
29	-11126.67	-720.56	617.04	-19892.39	-2384.84	2227.62
30	-11126.67	-700.44	722.57	-19892.39	-2200.96	2795.51
31	-11126.67	-671.42	826.17	-19892.39	-1977.22	3277.78
32	-11126.67	-630.20	926.34	-19892.39	-1738.23	3662.90
33	-11126.67	-579.44	1021.31	-19892.39	-1507.85	3960.63
34	-11126.67	-518.39	1111.70	-19892.39	-1238.45	4214.13
35	-11126.67	-447.15	1195.20	-19892.39	-1039.81	4368.97
36	-11126.67	-364.46	1268.31	-19892.39	-770.16	4517.62
37	-11126.67	-269.76	1325.50	-19892.39	-572.99	4603.18
38	-11126.67	-166.18	1367.41	-19892.39	-322.42	4665.74
39	-11126.67	-55.99	1391.86	-19892.39	-122.30	4694.04
40	-11126.67	56.98	1391.86	-19892.39	122.30	4694.04
41	-11126.67	167.13	1367.14	-19892.39	322.42	4665.74
42	-11126.67	270.60	1325.03	-19892.39	572.99	4603.18
43	-11126.67	365.22	1267.64	-19892.39	770.16	4517.62
44	-11126.67	447.84	1194.58	-19892.39	1039.81	4368.97
45	-11126.67	518.97	1110.97	-19892.39	1238.45	4214.13
46	-11126.67	579.90	1020.51	-19892.39	1507.85	3960.63
47	-11126.67	630.56	925.45	-19892.39	1738.23	3662.90
48	-11126.67	671.68	825.27	-19892.39	1977.22	3277.78
49	-11126.67	700.69	721.67	-19892.39	2200.96	2795.51
50	-11126.67	720.72	616.12	-19892.39	2384.84	2227.62
51	-11126.67	732.34	509.81	-19892.39	2511.26	1542.12
52	-11126.67	735.28	403.35	-19892.39	2557.77	811.22
53	-9686.97	1470.57	351.16	-17165.31	5075.23	836.95

Figure 12: Excel export: node coordinates and triangles.

Performance

The interaction surface computation is instantaneous in practice. The table below shows pure computation times (excluding GUI rendering) measured on a desktop PC for the two sections in this article, at three mesh resolutions (25×25, 50×50, and 100×100).

Section	Coarse (1.2k tri)	Medium (4.9k tri)	Fine (19.8k tri)
Octagonal – EC2	7 ms	17 ms	24 ms
Elliptical – ACI 318	7 ms	6 ms	10 ms

Eurocode 2 defines six limit states (three ULS, three SLS), each with its own material laws and admissible stresses, so the kernel produces six surfaces in a single call. ACI 318 uses two limit states (ULS and SLS), computed in two separate calls. All surfaces compute in under 25 ms even at the finest resolution.

Conclusion

The interaction surface provides a complete view of the resistance domain of a reinforced concrete section under biaxial bending. Rather than checking load points one at a time, the engineer can visualize the full capacity envelope and immediately assess whether a set of load combinations falls inside or outside the domain. This is exactly what the resistance verification module of SectionPro does: it projects each load combination onto the interaction surface and returns the safety margin, which will be the subject of the next article.

The two examples in this article illustrate how the geometry and the normative code shape the resistance domain. The octagonal section under Eurocode 2 and the elliptical section under ACI 318 both produce well-resolved 3D surfaces at interactive speed. The export formats (PDF, Excel, text) provide the data needed for calculation reports or external post-processing.