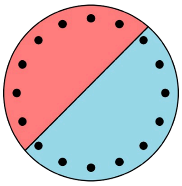


Resistance Check Based on Interaction Surfaces

SectionPro Tutorial: batch verification of load cases against the 3D resistance domain

BridgeKernel · 2026

Introduction

In the previous article, we computed the interaction surface, the 3D resistance domain of a reinforced concrete section in the (N, M_z, M_y) space. The stress-strain solver (Article #2) can verify individual loads against this domain, but the engineer must then inspect results one by one, or only look at the most unfavorable case, without a global picture of how all combinations sit relative to the capacity.

The distances module addresses this by projecting every load point onto the interaction surface and displaying the result as a single 3D scatter plot. For each load, it returns a status (inside, outside, or boundary) and a safety factor η quantifying the margin. The engineer sees all combinations at once: one glance reveals which loads are safe, which exceed the capacity, and by how much.

An additional advantage concerns codes with equivalent rectangular stress blocks (ACI 318 Whitney block, CSA A23.3, AASHTO). The stress-strain solver must use the alternative realistic law (parabola-rectangle), since a stress block cannot drive an iterative strain solver. The interaction surface, however, is built directly from the Whitney block, making the distances approach more faithful to these codes' primary design law.

The trade-off: unlike the stress-strain solver, the distances module does not return the deformation state or stress distribution. It answers "pass or fail, and by how much?" but not "what is the stress at each fiber?".

Computed results

SectionPro reports three categories of results for each distance analysis:

Status & safety factor

η: normalized distance
Status: Internal External
Most unfavorable load identified
One surface per limit state

3D visualization

Interaction surface (triangulated mesh)
Load points scattered on the plot
Color-coded by status
Rotation, zoom, pan controls

Exports

PDF: 3D views + results table
XLS: loads, distances, status
TXT: tabular results (columns)

This approach vs. the stress-strain analysis

The following table summarizes the key differences between the two verification methods available in SectionPro.

Criterion	Distances (this article)	Stress-strain (Article #2)
Goal	Pass/fail screening	Detailed state
Output	η + status	σ, ε, FS, forces
Deformation state	No	Yes
Visual output	3D scatter	Stress/strain diagrams
Best for	Large load envelopes	Critical load cases
Whitney block	Recommended	Use the realistic law
Few loads	Surface overhead	Fast (direct solve)
Many loads	Fast (one surface, cheap rays)	Slow (iterative per load)

Both approaches are complementary. A typical workflow is: (1) use distances to screen an entire load envelope and identify the critical combinations, then (2) use the stress-strain solver on those critical cases to obtain the full section response.

How distances work

Given a load point $P = \left(N, M_z, M_y\right)$ and the interaction surface $\mathcal{S}$, the module computes the centroid O of the surface mesh (guaranteed to lie inside the domain) and traces a ray from O through P until it intersects $\mathcal{S}$ at a point Q. The safety factor is defined as:

$$\eta = \frac{\|\overrightarrow{OP}\|}{\|\overrightarrow{OQ}\|}$$

The interpretation is straightforward:

- $\eta < 1$: the load point is **inside** the surface; the section has reserve capacity.
- $\eta = 1$: the load point is **on the boundary** the section is at its exact limit.
- $\eta > 1$: the load point is **outside** the surface; the capacity is exceeded.

On the 3D scatter plot, load points are color-coded accordingly: **green** for internal loads ($\eta < 1$) and **red** for external loads ($\eta > 1$).

The surface is computed once per limit state, and then each load point only requires a ray-surface intersection with negligible cost compared to the iterative convergence required by the stress-strain solver.

Octagonal section (Eurocode 2)

Input data

The section geometry, reinforcement, and material laws are identical to those used in Article #4 (Interaction Surface). 30 load combinations are defined: 15 at ULS-F (Fundamental) and 15 at SLS-C (Characteristic), covering a mix of pure axial force, pure biaxial bending, combined loading, tension, and compression.

Concrete

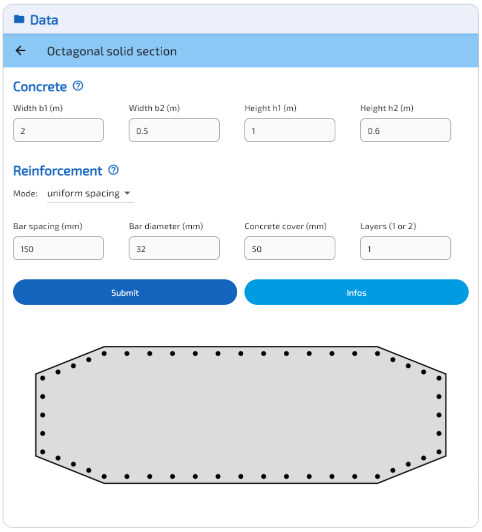
- Octagonal cross section
- $b_1 = 2.00$ m, $b_2 = 0.50$ m
- $h_1 = 1.00$ m, $h_2 = 0.60$ m

Reinforcement

- 48 bars, uniform spacing 150 mm
- Diameter $\varphi = 32$ mm, cover 50 mm

Material laws (EC2)

- Concrete C30/37: $f_{ck} = 30$ MPa
- Steel B500B: $f_{yk} = 500$ MPa



Width b1 (m): 2
Width b2 (m): 0.5
Height h1 (m): 1
Height h2 (m): 0.6

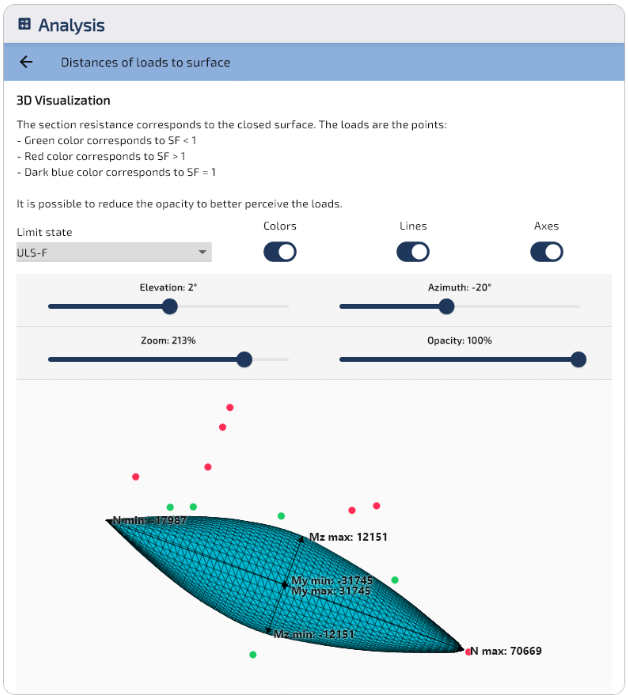
Mode: uniform spacing

Bar spacing (mm): 150
Bar diameter (mm): 32
Concrete cover (mm): 50
Layers (1 or 2): 1

Figure 1: Octagonal section: geometry and reinforcement layout.

ULS-F (Fundamental)

15 load combinations: 8 internal, 7 external.

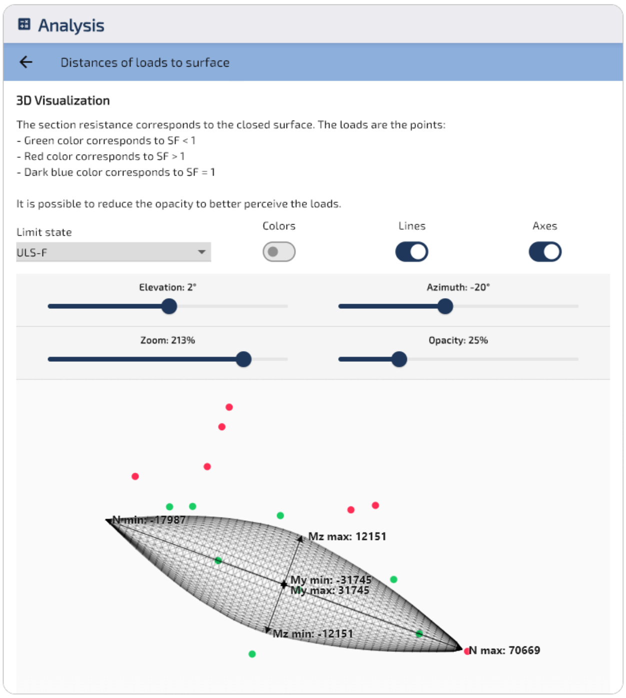


The section resistance corresponds to the closed surface. The loads are the points:
- Green color corresponds to SF < 1
- Red color corresponds to SF > 1
- Dark blue color corresponds to SF = 1

It is possible to reduce the opacity to better perceive the loads.

Limit state: ULS-F
Colors: on
Lines: on
Axes: on
Elevation: 2°
Azimuth: -20°
Zoom: 213%
Opacity: 100%

Figure 2: ULS-F: scattered loads on surface (view 1).



The section resistance corresponds to the closed surface. The loads are the points:
- Green color corresponds to SF < 1
- Red color corresponds to SF > 1
- Dark blue color corresponds to SF = 1

It is possible to reduce the opacity to better perceive the loads.

Limit state: ULS-F
Colors: off
Lines: on
Axes: on
Elevation: 2°
Azimuth: -20°
Zoom: 213%
Opacity: 25%

Figure 3: ULS-F: scattered loads on surface (view 2).

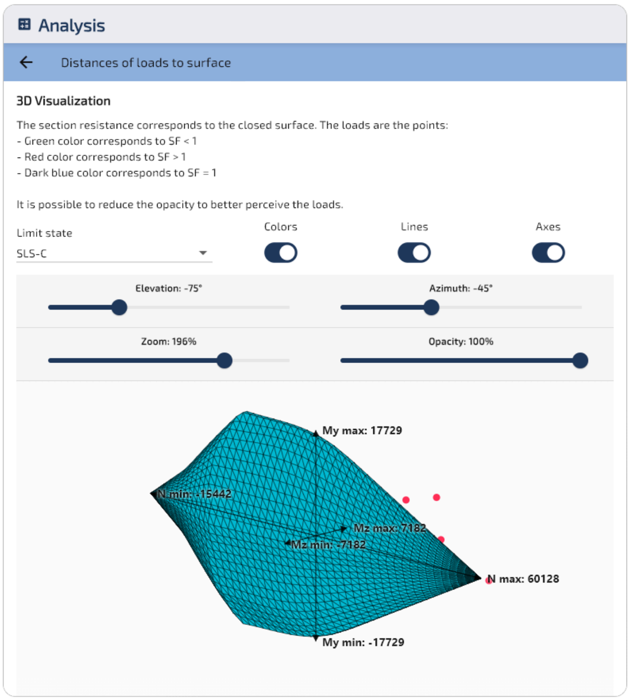
Load	N (kN)	M_z (kN·m)	M_y (kN·m)	η (−)	Status
8	0	14 000	35 000	1.879	External
7	0	11 000	30 000	1.605	External
4	72 000	0	0	1.030	External
5	0	5 000	10 000	0.916	Internal
3	60 000	0	0	0.761	Internal
2	30 000	0	0	0.088	Internal

Load #4 ($N = 72000$ kN, pure compression) barely exceeds the surface with $r_{\text{norm}} = 1.030$, confirming that the bounding box $N_{\max} = 70669$ kN reported in Article #4 is correct. Load #2 ($N = 30000$ kN, pure compression) is deep inside the domain ($r_{\text{norm}} = 0.088$), as expected for a load well below $N_{\max}$.

The combined loads show the non-cubic shape of the interaction surface: load #8 ($M_z = 14000$, $M_y = 35000$) has individual moment components below the bounding box limits ($M_{z,\max} = 12154$, $M_{y,\max} = 32317$) but their combination pushes the point outside the surface ($r_{\text{norm}} = 1.879$).

SLS-C (Characteristic)

15 load combinations: 6 internal, 9 external.



Analysis

← Distances of loads to surface

3D Visualization

The section resistance corresponds to the closed surface. The loads are the points:
- Green color corresponds to SF < 1
- Red color corresponds to SF > 1
- Dark blue color corresponds to SF = 1

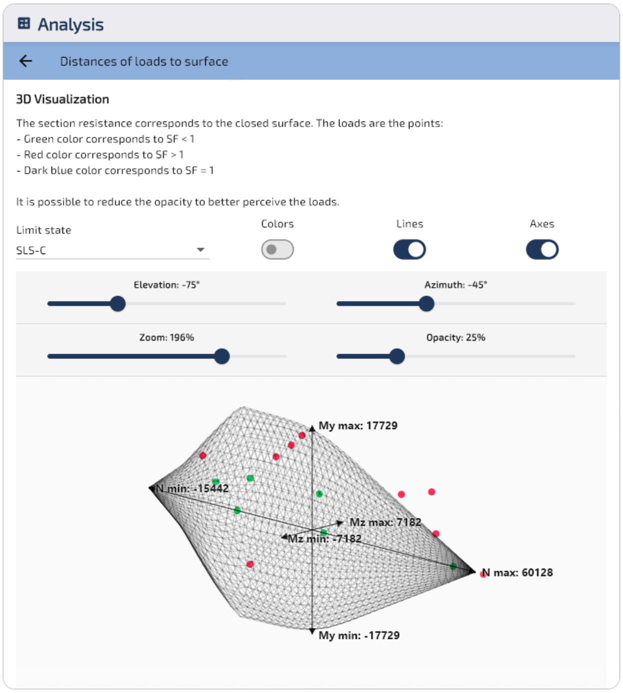
It is possible to reduce the opacity to better perceive the loads.

Limit state: SLS-C

Colors ●
Lines ●
Axes ●

Elevation: -75°
Azimuth: -45°
Zoom: 196%
Opacity: 100%

Figure 4: SLS-C: scattered loads on surface (view 1).



Analysis

← Distances of loads to surface

3D Visualization

The section resistance corresponds to the closed surface. The loads are the points:
- Green color corresponds to SF < 1
- Red color corresponds to SF > 1
- Dark blue color corresponds to SF = 1

It is possible to reduce the opacity to better perceive the loads.

Limit state: SLS-C

Colors ○
Lines ●
Axes ●

Elevation: -75°
Azimuth: -45°
Zoom: 196%
Opacity: 25%

Figure 5: SLS-C: scattered loads on surface (view 2).

Load	N (kN)	M_z (kN·m)	M_y (kN·m)	η (−)	Status
23	0	8 000	20 000	1.910	External
26	35 000	6 000	15 000	1.795	External
19	62 000	0	0	1.050	External
27	−5 000	2 000	5 000	0.990	Internal
18	55 000	0	0	0.866	Internal
17	25 000	0	0	0.081	Internal

NB: *To better identify internal load points hidden behind the surface, reduce the surface opacity or switch to wireframe mode (both options are available in the 3D viewer).*

Elliptical section (ACI 318)

Input data

The section geometry, reinforcement, and material laws are identical to those used in Article #4. 30 load combinations are defined: 15 at ULS and 15 at SLS.

Concrete

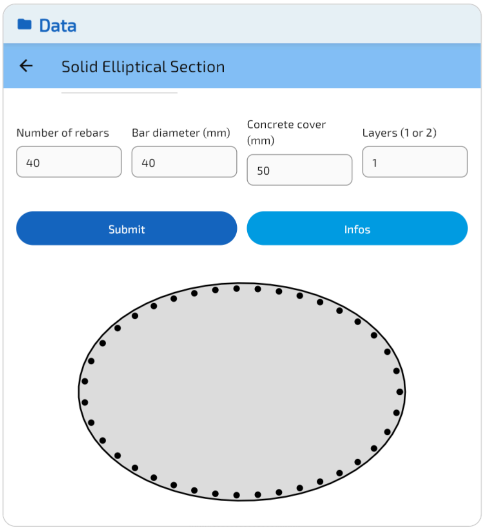
- Elliptical cross section
- Width = 3.00 m, Height = 2.00 m

Reinforcement

- 40 bars along the perimeter
- Diameter $\varphi = 40$ mm, cover 50 mm

Material laws (ACI 318)

- Concrete: $f'_c = 30$ MPa
- Steel: $f_y = 500$ MPa



Data

Solid Elliptical Section

Number of rebars: 40

Bar diameter (mm): 40

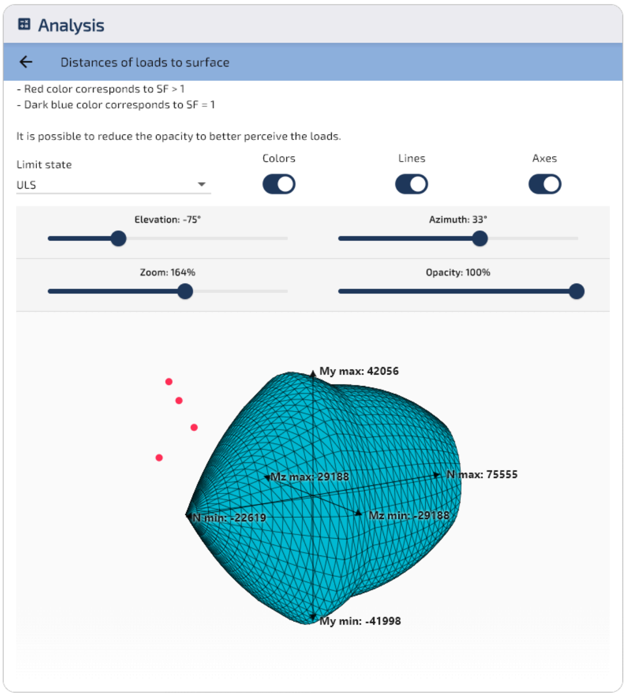
Concrete cover (mm): 50

Layers (1 or 2): 1

Figure 6: Elliptical section: geometry and reinforcement.

ULS

15 load combinations: 8 internal, 7 external.



Analysis

Distances of loads to surface

- Red color corresponds to SF > 1
- Dark blue color corresponds to SF = 1

It is possible to reduce the opacity to better perceive the loads.

Limit state: ULS

Colors: on; Lines: on; Axes: on

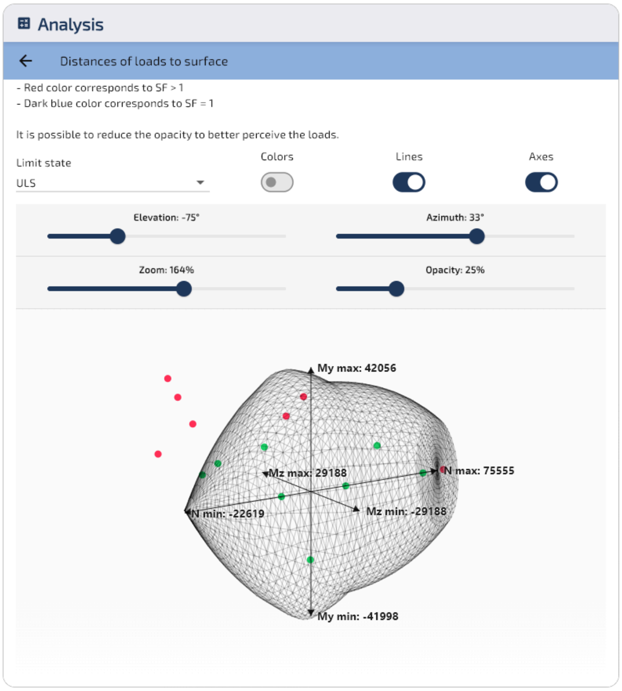
Elevation: -75°

Azimuth: 33°

Zoom: 164%

Opacity: 100%

Figure 7: ULS: scattered loads on surface (view 1).



Analysis

Distances of loads to surface

- Red color corresponds to SF > 1
- Dark blue color corresponds to SF = 1

It is possible to reduce the opacity to better perceive the loads.

Limit state: ULS

Colors: off; Lines: on; Axes: on

Elevation: -75°

Azimuth: 33°

Zoom: 164%

Opacity: 25%

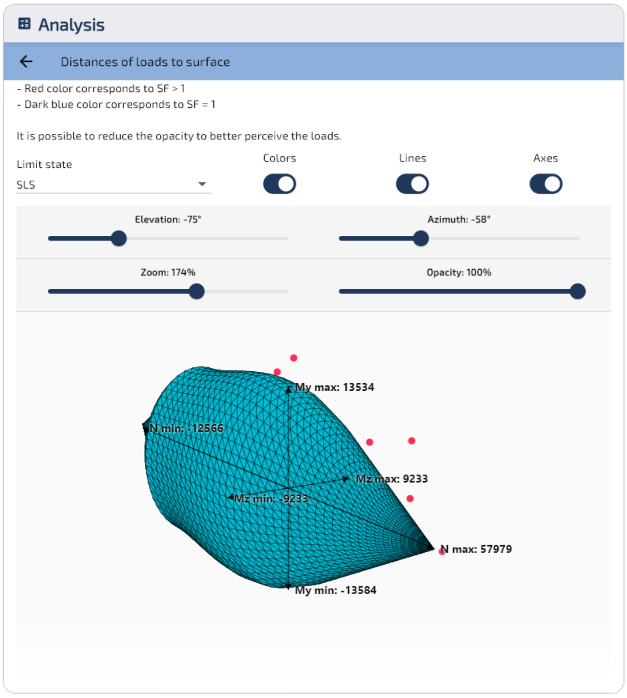
Figure 8: ULS: scattered loads on surface (view 2).

Load	N (kN)	M_z (kN·m)	M_y (kN·m)	η (−)	Status
8	0	32 000	45 000	1.646	External
7	0	27 000	39 000	1.475	External
4	78 000	0	0	1.067	External
15	10 000	−15 000	−25 000	0.967	Internal
3	70 000	0	0	0.847	Internal
2	40 000	0	0	0.022	Internal

The ACI φ-factors ($\varphi = 0.65$ to 0.90) and the cap $\varphi_N = 0.80$ reduce the nominal capacity, making the ULS surface smaller than a raw interaction surface. From Article #4, the bounding box gives $N_{\max} = 75555$ kN, $M_{z,\max} = 29188$ kN · m, $M_{y,\max} = 42056$ kN · m: exceeding any of these limits guarantees failure, as seen for loads #4 and #8. Load #7 ($M_z = 27000$, $M_y = 39000$ kN · m), however, stays within all three limits yet still falls outside the surface ($\eta = 1.475$) — the bounding box cannot catch this case, the 3D surface can.

SLS

15 load combinations: 7 internal, 8 external.



- Red color corresponds to SF > 1
- Dark blue color corresponds to SF = 1

It is possible to reduce the opacity to better perceive the loads.

Limit state: SLS

Colors ● Lines ● Axes ●

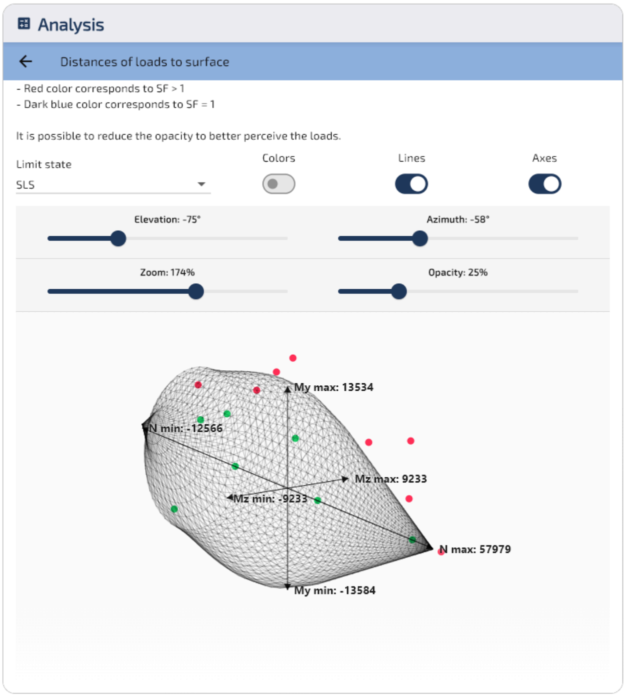
Elevation: -75°

Azimuth: -58°

Zoom: 174%

Opacity: 100%

Figure 9: SLS: scattered loads on surface (view 1).



- Red color corresponds to SF > 1
- Dark blue color corresponds to SF = 1

It is possible to reduce the opacity to better perceive the loads.

Limit state: SLS

Colors ○ Lines ● Axes ●

Elevation: -75°

Azimuth: -58°

Zoom: 174%

Opacity: 25%

Figure 10: SLS: scattered loads on surface (view 2).

Load	N (kN)	M_z (kN·m)	M_y (kN·m)	η (−)	Status
23	0	10 000	15 000	1.688	External
26	35 000	7 500	11 000	1.487	External
19	60 000	0	0	1.068	External
27	−4 000	2 500	3 500	0.942	Internal
18	53 000	0	0	0.868	Internal
17	30 000	0	0	0.216	Internal

At SLS, the concrete is limited to the allowable stress ($\sigma_c = 11.5$ MPa), resulting in a much smaller surface than ULS. Load #23 is the most unfavorable across both limit states ($r_{\text{norm}} = 1.688$): the combined biaxial bending ($M_z = 10000$, $M_y = 15000$ kN·m) far exceeds the SLS capacity, even though each moment component individually would be within the bounding box.

Cross-validation with the stress-strain solver

The distances module projects load points onto a pre-built mesh of the interaction surface. The stress-strain solver (Newton-Raphson, Article #2) iterates to find the equilibrium strain state for each load individually. The two methods should agree: a load inside the surface ($\eta < 1$) must satisfy all material strain limits, while a load outside ($\eta > 1$) must violate at least one limit.

15-load comparison (octagonal section, ULS-F)

For each load, the table gives the distances result (η and Internal/External status), followed by the stress-strain solver output: worst concrete strain ε_c and steel strain ε_s (both in ‰, absolute values), and the corresponding material verdict.

Load	N (kN)	M_z (kN·m)	M_y (kN·m)	η (–)	Status	ε_c (‰)	ε_s (‰)	Verdict
1	10 000	0	0	0.390	Internal	0.17	0.17	OK
2	30 000	0	0	0.072	Internal	0.54	0.54	OK
3	60 000	0	0	0.757	Internal	1.36	1.36	OK
4	72 000	0	0	1.031	External	2.17	2.17	KO
5	0	5 000	10 000	0.920	Internal	1.50	3.13	OK
6	0	8 000	20 000	1.224	External	18.0	73.6	KO
7	0	11 000	30 000	1.584	External	26.2	112	KO
8	0	14 000	35 000	1.864	External	33.3	140	KO
9	20 000	5 000	15 000	0.691	Internal	1.66	1.58	OK
10	35 000	8 000	22 000	1.108	External	4.65	4.43	KO
11	40 000	10 000	25 000	1.365	External	6.93	6.59	KO
12	−5 000	3 000	8 000	0.912	Internal	0.86	2.89	OK
13	−15 000	5 000	12 000	1.260	External	11.5	97.9	KO
14	50 000	4 000	10 000	0.847	Internal	2.22	2.15	OK
15	25 000	−6 000	−18 000	0.813	Internal	2.19	2.09	OK

The two methods are fully consistent. Every External load is confirmed at failure by at least one material (concrete, steel, or both), and every Internal load satisfies all strain limits. The safety factor is a reliable indicator of margin: loads deep inside the surface show strains well below their limits, while loads near the boundary approach them and loads well outside exceed them by a large margin. Loads 10–11: concrete crushing only, steel within rupture limit. Loads 6–8 and 13: both limits exceeded simultaneously.

Note: As explained in Article #2, the Newton-Raphson solver extrapolates material laws beyond their physical validity domain when equilibrium cannot be reached within the valid range. Strains reported for External loads are therefore numerical artefacts with no physical meaning: they simply confirm that no valid equilibrium state exists inside the material limits.

100 000-load benchmark

To quantify agreement at scale, both methods are applied to 100 000 random load combinations ($N \in [-20\,000, 80\,000]$ kN, $M_z, M_y \in [-50\,000, 50\,000]$ kN · m, all at ULS-F). The surface is built once (31 ms) and reused for all distance queries.

Method	Loads	Query time	Rate	Internal	External
Distances (queries only)	100 000	13 ms	7.5 M/s	5.1%	94.9%
Stress-strain NR	100 000	207 ms	0.48 M/s	5.1%	94.9%

Agreement: **99.97%** (99 974 out of 100 000 loads classified identically). The 26 disagreements all have $|\eta - 1| < 0.002$: these load points lie within 0.2% of the surface boundary and are effectively **at the limit** by any measure.

This is expected behaviour. The distances module does not apply a strict equality test $\eta = 1$: any load with η sufficiently close to 1 is treated as a boundary case. In this narrow region, the two methods can legitimately disagree — the distances result depends on the mesh resolution (finite triangle size introduces a geometric approximation), while the NR solver iterates to exact equilibrium. In such cases the NR solver is the final arbiter: it computes exact equilibrium and its verdict takes precedence over the distances classification.

From an engineering standpoint, whenever $\eta \approx 1$, the engineer should not rely on the automatic Internal/External classification alone. The appropriate response is either to run a full NR calculation for a precise verdict, or, better, to modify the section geometry or reinforcement to achieve a clear safety margin (η comfortably below 1).

The distances module is **15 times faster** than the NR solver for this batch (query phase). In practice, however, both methods are effectively instantaneous for the vast majority of engineering use cases. The speed advantage becomes meaningful for advanced applications (structural optimisation loops, parametric studies, automated code-checking over large load envelopes) where millions of combinations or more must be evaluated repeatedly.

Conclusion

The distances module provides a fast and reliable method to screen any number of load combinations against the interaction surface of a reinforced concrete section. For each load, it returns a normalised safety factor η and an Internal/External status, giving the engineer an immediate picture of the most critical combinations across all limit states simultaneously.

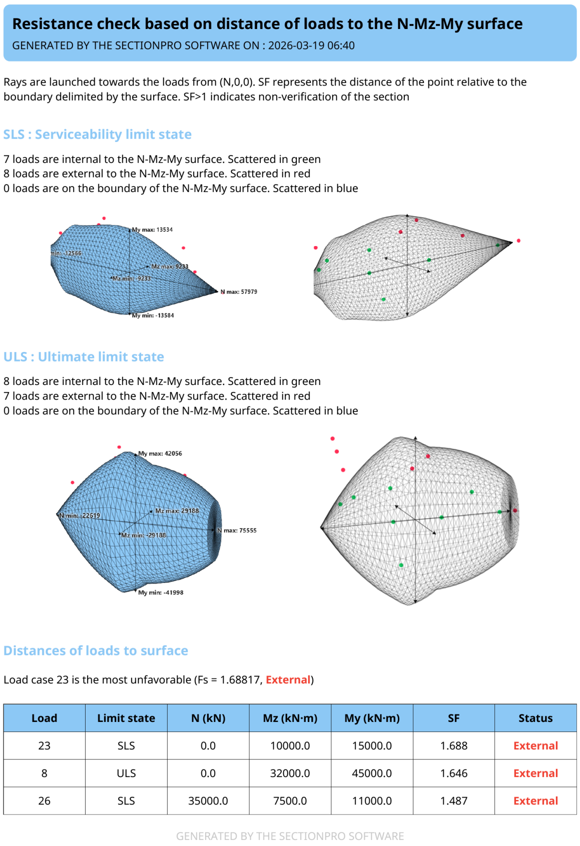
The cross-validation on 100 000 loads confirms 99.97% agreement with the Newton-Raphson stress-strain solver. The 26 disagreements are all located within 0.2% of the surface boundary, where mesh discretisation makes the classification uncertain; in these cases the NR solver remains the final arbiter. For loads clearly inside or outside the surface, the two methods are fully consistent.

Both methods are instantaneous for routine engineering work. The distances approach becomes especially valuable when millions of combinations must be evaluated (optimisation loops, parametric studies, automated code-checking), where its surface-reuse architecture eliminates redundant computation entirely.

Beyond the numerical results, the key advantage of the distances module is the 3D scatter plot: all load combinations, all limit states, and the full resistance domain are visible in a single figure. At a glance, the engineer sees which loads are safe, which exceed capacity, and by how much, a self-contained graphic that integrates directly into a calculation report.

Export

SectionPro exports the distance results in three formats. The PDF report includes 3D views of the interaction surface with scattered load points. For each limit state, the most critical load is identified, followed by a results table sorted by descending η. The Excel and text exports provide the same tabular data for external post-processing.



Resistance check based on distance of loads to the N-Mz-My surface

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Rays are launched towards the loads from (N,0,0). SF represents the distance of the point relative to the boundary delimited by the surface. SF>1 indicates non-verification of the section

SLS : Serviceability limit state

7 loads are internal to the N-Mz-My surface. Scattered in green
8 loads are external to the N-Mz-My surface. Scattered in red
0 loads are on the boundary of the N-Mz-My surface. Scattered in blue

ULS : Ultimate limit state

8 loads are internal to the N-Mz-My surface. Scattered in green
7 loads are external to the N-Mz-My surface. Scattered in red
0 loads are on the boundary of the N-Mz-My surface. Scattered in blue

Distances of loads to surface

Load case 23 is the most unfavorable (Fs = 1.68817, External)

Load	Limit state	N (kN)	Mz (kN·m)	My (kN·m)	SF	Status
23	SLS	0.0	10000.0	15000.0	1.688	External
8	ULS	0.0	32000.0	45000.0	1.646	External
26	SLS	35000.0	7500.0	11000.0	1.487	External

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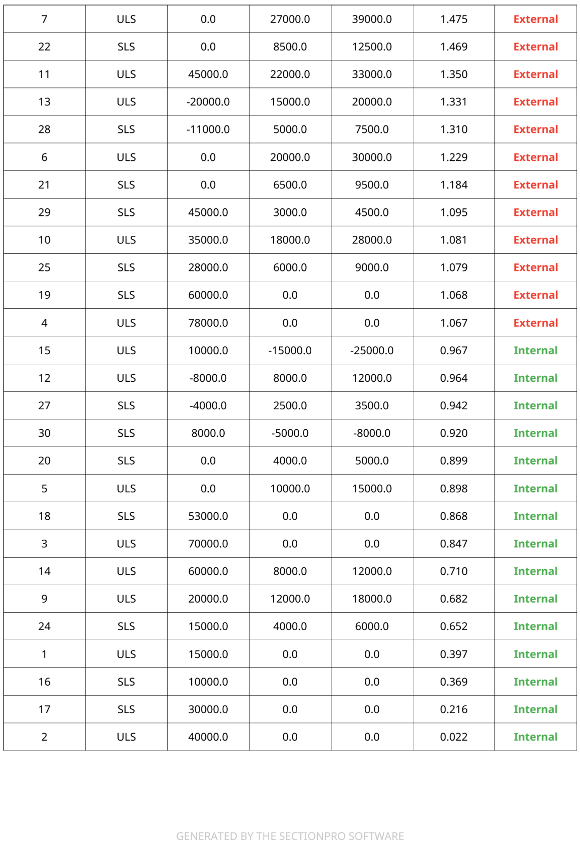
Figure 11: PDF export, page 1: 3D views and load scatter.



7	ULS	0.0	27000.0	39000.0	1.475	External
22	SLS	0.0	8500.0	12500.0	1.469	External
11	ULS	45000.0	22000.0	33000.0	1.350	External
13	ULS	-20000.0	15000.0	20000.0	1.331	External
28	SLS	-11000.0	5000.0	7500.0	1.310	External
6	ULS	0.0	20000.0	30000.0	1.229	External
21	SLS	0.0	6500.0	9500.0	1.184	External
29	SLS	45000.0	3000.0	4500.0	1.095	External
10	ULS	35000.0	18000.0	28000.0	1.081	External
25	SLS	28000.0	6000.0	9000.0	1.079	External
19	SLS	60000.0	0.0	0.0	1.068	External
4	ULS	78000.0	0.0	0.0	1.067	External
15	ULS	10000.0	-15000.0	-25000.0	0.967	Internal
12	ULS	-8000.0	8000.0	12000.0	0.964	Internal
27	SLS	-4000.0	2500.0	3500.0	0.942	Internal
30	SLS	8000.0	-5000.0	-8000.0	0.920	Internal
20	SLS	0.0	4000.0	5000.0	0.899	Internal
5	ULS	0.0	10000.0	15000.0	0.898	Internal
18	SLS	53000.0	0.0	0.0	0.868	Internal
3	ULS	70000.0	0.0	0.0	0.847	Internal
14	ULS	60000.0	8000.0	12000.0	0.710	Internal
9	ULS	20000.0	12000.0	18000.0	0.682	Internal
24	SLS	15000.0	4000.0	6000.0	0.652	Internal
1	ULS	15000.0	0.0	0.0	0.397	Internal
16	SLS	10000.0	0.0	0.0	0.369	Internal
17	SLS	30000.0	0.0	0.0	0.216	Internal
2	ULS	40000.0	0.0	0.0	0.022	Internal

GENERATED BY THE SECTIONPRO SOFTWARE

Figure 12: PDF export, page 2: results table.